INTRODUCTION TO LOGIC

3 Formalisation in Propositional Logic

Volker Halbach

If I could choose between principle and logic,
I'd take principle every time.

Maggie Smith as Violet Crawley
in Downton Abbey series 6, ep. 6

Last week I introduced the formal language $\mathcal{L}_1$ of propositional logic.

This week I'll relate $\mathcal{L}_1$ to English. I'll discuss how one can translate English sentences into $\mathcal{L}_1$-sentences and how the notions of validity etc. in $\mathcal{L}_1$ and English are related.

I’ll explain how to translate English sentences into sentences of $\mathcal{L}_1$.

I'll explain how to translate English sentences into sentences of $\mathcal{L}_1$.

This involves two steps:

1. The English sentence is brought into a standardised form, the logical form.
2. The English connectives are replaced with connectives of $\mathcal{L}_1$, the remaining English sentences are replaced with sentence letters according to a dictionary that is specified.

In the *Manual* I sketch a general procedure for generating the logical form of a sentence; here I give only an example.

In the process of finding the logical form, one should reformulate English sentences in such a way that they are built up using the standard connectives:

name	standard connective	some other formulations
conjunction	and	but, although, [comma]
disjunction	or	unless
negation	it is not the case that	not, none, never
arrow	if … then	provided that, if… only if (reversed order)
double arrow	if and only if	exactly if, precisely if

Example

If Hamilton finishes in the top 10 or Vettel doesn't win, Hamilton is world champion.

This is the English sentence.

Example

If Hamilton finishes in the top 10 or Vettel doesn't win, Hamilton is world champion.

It is structured like this. 'if' is its main connective.

Example

If Hamilton finishes in the top 10 or Vettel doesn't win, then Hamilton is world champion.

The standard connective is 'if … then' (not just 'if').

Example

(If Hamilton finishes in the top 10 or Vettel doesn't win, then Hamilton is world champion)

Now that we have sentence with a standard connective, the entire sentence is put into brackets…

Example

(If Hamilton finishes in the top 10 or Vettel doesn't win, then Hamilton is world champion)

… and I repeat the procedure with the sentences that make up the entire sentence. First I turn to the blue sentence (order doesn't matter).

Example

(If (Hamilton finishes in the top 10 or Vettel doesn't win), then Hamilton is world champion)

'or' is a standard connective. So I enclose the blue sentence in brackets.

Example

(If ((Hamilton finishes in the top 10) or Vettel doesn't win), then Hamilton is world champion)

The sentence 'Hamilton finishes in the top 10' cannot be further analysed; I enclose it in brackets and turn to the red sentence, which is a negated sentence

Example

(If ((Hamilton finishes in the top 10) or it is not the case that Vettel wins), then Hamilton is world champion)

… and can be reformulated with the standard connective ‘it is not the case that’.

Example

(If ((Hamilton finishes in the top 10) or it is not the case that (Vettel wins)), then Hamilton is world champion)

The red sentence cannot be further analysed and it put in brackets.

Example

(If ((Hamilton finishes in the top 10) or it is not the case that (Vettel wins)), then (Hamilton is world champion))

The blue sentence cannot be further analysed; so it's enclosed in brackets and not further analysed…

Example

(If ((Hamilton finishes in the top 10) or it is not the case that (Vettel wins)), then (Hamilton is world champion))

…and we have arrived at the logical form of the sentence.

This was very detailed.

You are not expected to learn the steps of the procedure by heart, but you need to be able to determine the logical form of a given sentence.

Going from the logical form to the formalisation in $\mathcal{L}_1$ involves three steps:

1. Replace standard connectives by their respective symbols in accordance with the following list:

standard connective	symbol
and	$\wedge$
or	$\vee$
it is not the case that	$\neg$
if … then …	$\rightarrow$
if and only if	$\leftrightarrow$

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2. Replace every English sentence by a sentence letter and delete the brackets surrounding the sentence letter.

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2. Replace every English sentence by a sentence letter and delete the brackets surrounding the sentence letter.
3. Provide a dictionary.

I'll explain the procedure using the example from above:

Example

If Hamilton finishes in the top 10 or Vettel doesn't win, Hamilton is world champion.

Example

(If ((Hamilton finishes in the top 10) or it is not the case that (Vettel wins)) , then (Hamilton is world champion))

This is the logical form we have already determined.

Example

(((Hamilton finishes in the top 10) $\vee$ $\neg$ (Vettel wins)) $\to$ (Hamilton is world champion))

First I replace all standard connectives by the respective symbols.

Example

$($ P $\lor$ $\neg$ Q $) \rightarrow$ R $)$

In step 2 the sentences are replace by sentence letters; the brackets surrounding the sentence letters are deleted.

Example

(P $\lor$ $\neg$
Q) $\to$ R)

Thus we have $((P \lor \neg Q) \to R)$ with the following dictionary:

P: Hamilton finishes in the top 10.
Q: Vettel wins.
R: Hamilton is world champion.

There are various problems with formalising English sentences in $\mathcal{L}_1$:

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(i) Some English connectives aren't truth-functional.
(ii) In some cases the logical form cannot be uniquely determined because the English sentence is ambiguous.
(iii) It's not clear how much 'force' may be applied in order to reformulate an English sentence as a sentence with a standard connective.

English also contains connectives that don’t correspond to standard connectives.

‘It could be the case that’

‘It must be the case that’

‘Volker thought that’

‘because’

‘logically entails that’

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'It could be the case that'

'It must be the case that'

'Volker thought that'

'because'

'logically entails that'

'Volker thought that' and 'Violet likes logic' make 'Volker thought that Violet likes logic'.

None of these connectives can be can be captured in $\mathcal{L}_1$.

Truth functionality

Only truth-functional connectives can be captured in $\mathcal{L}_1$.

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Example: a truth-functional connective

The truth-value of 'It is not the case that *A*' is fully determined by the truth-value of *A*.

A	It is not the case that A
T	F
F	T

Example: a non-truth-functional connective

The truth-value of 'It is possibly the case that *A*' is *not* fully determined by the truth-value of *A*.

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A_1 James Studd is giving this lecture.

A_2 Two plus two equals five.

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It is possibly the case that A_2.	

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Characterisation: truth-functional (p. 54)

A connective is *truth-functional* if and only if the truth-value of the compound sentence cannot be changed by replacing a direct subsentence with another having the same truth-value.

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NB: replacing non-direct subsentences may change the truth-value.

If … then

The connective 'if, …then' is usually translated as $\rightarrow$. It's surely not adequate for counterfactuals:

Example

If I hadn't given the logic lecture last week, David Cameron would have given it.

Some other cases are controversial:

Example

If I don't give the logic lecture next week, the UK will leave the EU next year.

ϕ	ψ	$(\phi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

So one might doubt that 'if,…then' is always truth functional.

Rules of thumb for $\rightarrow$

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Formalise:

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(i) If John revised, [then] he passed.

Rules of thumb for $\rightarrow$

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Dictionary: R: John revised. P: John passed.

Rules of thumb for $\rightarrow$

Formalise:

(i) If John revised, [then] he passed. $R \rightarrow P$

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(i) *Formalisation*: $R \rightarrow P$

Rules of thumb for $\rightarrow$

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(ii) John passed if he revised.

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Rules of thumb for $\to$

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(ii) Paraphrase: (i).

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Rules of thumb for $\rightarrow$

Formalise:

(i) If John revised, [then] he passed. $R \rightarrow P$

(ii) John passed if he revised. ‘$P \leftarrow R$’ i.e. $R \rightarrow P$

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Rules of thumb for $\rightarrow$

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(iii) John passed only if he revised.

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(iii) Paraphrase: If John passed, John revised.

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Ambiguity

Example

Locke is right and Reid’s argument is convincing or Hume is right.

Ambiguity

Example

Locke is right and Reid's argument is convincing or Hume is right.

Logical form: variant (i)

(((Locke is right) and (Reid's argument is convincing)) or (Hume is right))

Logical form: variant (ii)

((Locke is right) and ((Reid's argument is convincing) or (Hume is right)))

Ambiguity

Example

Locke is right and Reid's argument is convincing or Hume is right.

Logical form: variant (i)

(((Locke is right) and (Reid's argument is convincing)) or (Hume is right))

Logical form: variant (ii)

((Locke is right) and ((Reid's argument is convincing) or (Hume is right)))

(i) $(P \wedge Q) \vee R$

(ii) $P \wedge (Q \vee R)$, with the following dictionary

P: Locke is right

Q: Reid's argument is convincing

R: Hume is right

This is a case of scope ambiguity: it's not clear whether 'and' connects 'Locke is right' and 'Reid's argument is convincing' or whether it connects 'Locke is right' and 'Reid's argument is convincing or Hume is right'.

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(ii) $\underbrace{P \wedge (Q \vee R)}$

The underbraced part is the scope of (the occurrence of) $\wedge$.

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(i) $\underbrace{(P \land Q)} \lor R$

(ii) $\underbrace{P \land (Q \lor R)}$

The underbraced part is the scope of (the occurrence of) $\land$.

Definition (scope of a connective)

The scope of an occurrence of a connective in a sentence ϕ is (the occurrence of) the smallest subsentence of ϕ that contains this occurrence of the connective.

Reformulations

Example

Tom and Mary are tall.

Reformulations

Example

Tom is tall and Mary is tall.

This reformulation is ok.

Reformulations

Example

Tom is tall and Mary is tall.

This reformulation is ok.

Example

Tom and Mary are married.

Reformulations

Example

Tom is tall and Mary is tall.

This reformulation is ok.

Example

Tom is married and Mary is married.

This reformulation is arguably not correct.

Validity, logical truths etc.

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The sentence of $\mathcal{L}_1$ that is obtained by translating an English sentence into the language of propositional logic is the formalisation of that sentence.

As we have just seen, the formalisation may not be uniquely determined because of ambiguity (and also because one may use different sentence letters; but the choice of sentence letters doesn't matter for what follows).

Definition

(i) An English sentence is a tautology (propositionally valid) if and only if its formalisation in propositional logic is logically true (that is, iff it is a tautology).

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(ii) An English sentence is a propositional contradiction if and only if its formalisation in propositional logic is a contradiction.

(iii) An argument in English is propositionally valid if and only if its formalisation in $\mathcal{L}_1$ is valid.

In order to check whether an argument is propositionally valid, one can proceed in the following way:

1. Formalise all sentences in the argument in $\mathcal{L}_1$.
2. Check whether the resulting $\mathcal{L}_1$-argument is valid (e.g. using a truth table).

For tautologies and propositional contradiction we proceed in a similar way.

I show that the following argument is propositionally valid.

Example

Either CO_2-emissions are being cut or there will be more floods. It is not the case that CO_2-emissions are being cut. Therefore there will be more floods.

I'll deal with the sentences one by one.

FIRST PREMISS

Either CO_2-emissions are being cut or there will be more floods.

FIRST PREMISS

Either CO_2-emissions are being cut or there will be more floods.

LOGICAL FORM

((CO_2-emissions are being cut) or (there will be more floods))

I'm not quite sure whether 'either or' is exclusive, but I think it isn't.

FORMALISATION

$(P \vee Q)$

P: CO_2-emissions are being cut
Q: there will be more floods

SECOND PREMISS

It is not the case that CO_2-emissions are being cut.

SECOND PREMISS

It is not the case that CO_2-emissions are being cut.

LOGICAL FORM

It is not the case that (CO_2-emissions are being cut)

SECOND PREMISS

It is not the case that CO_2-emissions are being cut.

LOGICAL FORM

It is not the case that (CO_2-emissions are being cut)

FORMALISATION

$\neg P$

Of course I must use the same dictionary:

P: CO_2-emissions are being cut

Q: there will be more floods

CONCLUSION

There will be more floods.

CONCLUSION

There will be more floods.

LOGICAL FORM

(There will be more floods)

CONCLUSION

There will be more floods.

LOGICAL FORM

(There will be more floods)

FORMALISATION

Q

P: CO_2-emissions are being cut

Q: there will be more floods

So the premisses are formalised as $(P \vee Q)$ and $\neg P$ and the conclusion as Q.

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Next I show that the corresponding $\mathcal{L}_1$-argument is valid:

$$(P \vee Q), \neg P \vDash Q$$

So the premisses are formalised as $(P \lor Q)$ and $\neg P$ and the conclusion as Q.

Next I show that the corresponding $\mathcal{L}_1$-argument is valid:

$$(P \lor Q), \neg P \vDash Q$$

P	Q	$(P \lor Q)$	$\neg P$	Q

I want to show the claim using a partial truth table.

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Next I show that the corresponding $\mathcal{L}_1$-argument is valid:

$$(P \vee Q), \neg P \vDash Q$$

P	Q	$(P \vee Q)$	$\neg\, P$	Q
		T	T	F

I want to show the claim using a partial truth table. I assume that there is a $\mathcal{L}_1$-structure (i.e. a line in the truth table that makes all premisses true and the conclusion false).

So the premisses are formalised as $(P \vee Q)$ and $\neg P$ and the conclusion as Q.

Next I show that the corresponding $\mathcal{L}_1$-argument is valid:

$$(P \vee Q), \neg P \vDash Q$$

P	Q	$(P \vee Q)$	$\neg P$	Q
		T F	T	F

I want to show the claim using a partial truth table. I assume that there is a $\mathcal{L}_1$-structure (i.e. a line in the truth table that makes all premisses true and the conclusion false). Since Q is false and $P \vee Q$ is true,

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Thus, the English argument is propositionally valid.

The following sentence is a tautology:

Example

If Violet doesn't like logic, then she is stubborn or she doesn't like logic.

FORMALISATION

$P \to (Q \vee P)$

P: Violet doesn't like logic
Q: Violet is stubborn

Here we don't need a full formalisation.

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An English argument is propositionally valid independently of the 'meaning' of the sentences that are replaced by sentence letters.

Analogous remarks apply to logical truths and propositional contradictions.

There are arguments that are *logically* valid without being *propositionally* valid.

Example

Zeno is a tortoise. All tortoises are toothless. Therefore Zeno is toothless.

	P:	Zeno is a tortoise.
Dictionary	Q:	All tortoises are toothless.
	R:	Zeno is toothless.

But the argument with P and Q as premisses and Q as conclusion isn't valid, while the English argument is.

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We need a more powerful formal language: $\mathcal{L}_2$.