

INTRODUCTION TO LOGIC

2 Syntax and Semantics of Propositional Logic

Volker Halbach

Logic is the beginning of wisdom.

Thomas Aquinas

In what follows I look at some formal languages that are *much* simpler than English and define *validity of arguments*, ‘truth under an interpretation’, *consistency* etc. for these formal languages.

In what follows I look at some formal languages that are *much* simpler than English and define *validity of arguments*, ‘truth under an interpretation’, *consistency* etc. for these formal languages.

In logic one abstracts from all stylistic variants etc of natural language and retains just the basic skeleton of the language in a regimented form.

When presenting a formal language, I proceed in the following order:

- 1 I specify the **syntax** or grammar of the language; in particular I define what the sentences of the language are.

When presenting a formal language, I proceed in the following order:

- ① I specify the **syntax** or grammar of the language; in particular I define what the sentences of the language are.
- ② I specify the **semantics** of the language; in particular, I say what it means for a sentence to be true under an interpretation (or in a 'structure'). Once the notion of an interpretation (or structure) is clear, I can define validity of arguments etc as for English.

Syntax is all about *expressions*: words and sentences.

Syntax is all about *expressions*: words and sentences.

Examples of syntactic claims

- 'Bertrand Russell' is a proper noun.
- 'likes logic' is a verb phrase.
- 'Bertrand Russell likes logic' is a sentence.
- Combining a proper noun and a verb phrase in this way yields a sentence.

Semantics is all about *meanings* of expressions.

Semantics is all about *meanings* of expressions.

Examples of semantic claims

- 'Bertrand Russell' refers to a British philosopher.
- 'Bertrand Russell' refers to Bertrand Russell.
- 'likes logic' expresses a property Russell has.
- 'Bertrand Russell likes logic' is true.

Note our use of quotes to talk about expressions.

Note our use of quotes to talk about expressions.

'Bertrand Russell' refers to Bertrand Russell.

Note our use of quotes to talk about expressions.

'Bertrand Russell' refers to Bertrand Russell.

Mention

- The first occurrence of 'Bertrand Russell' is an example of mention.

Note our use of quotes to talk about expressions.

'Bertrand Russell' refers to Bertrand Russell.

Mention

- The first occurrence of 'Bertrand Russell' is an example of mention.
- This occurrence (with quotes) refers to an expression.

Note our use of quotes to talk about expressions.

'Bertrand Russell' refers to **Bertrand Russell**.

Mention

- The first occurrence of 'Bertrand Russell' is an example of mention.
- This occurrence (with quotes) refers to an expression.

Use

- The second occurrence of 'Bertrand Russell' is an example of use.

Note our use of quotes to talk about expressions.

‘Bertrand Russell’ refers to **Bertrand Russell**.

Mention

- The first occurrence of ‘Bertrand Russell’ is an example of mention.
- This occurrence (with quotes) refers to an expression.

Use

- The second occurrence of ‘Bertrand Russell’ is an example of use.
- This occurrence (without quotes) refers to a man.

Syntax: English vs. \mathcal{L}_1 .

English has *many* different sorts of expressions.

Syntax: English vs. \mathcal{L}_1 .

English has *many* different sorts of expressions.

Some expressions of English

Syntax: English vs. \mathcal{L}_1 .

English has *many* different sorts of expressions.

Some expressions of English

- (1) *Sentences*: 'Bertrand Russell likes logic', 'Philosophers like conceptual analysis', etc..

Syntax: English vs. \mathcal{L}_1 .

English has *many* different sorts of expressions.

Some expressions of English

- (1) *Sentences*: ‘Bertrand Russell likes logic’, ‘Philosophers like conceptual analysis’, etc..
- (2) *Connectives*: ‘it is not the case that’, ‘and’, etc..

Syntax: English vs. \mathcal{L}_1 .

English has *many* different sorts of expressions.

Some expressions of English

- (1) *Sentences*: ‘Bertrand Russell likes logic’, ‘Philosophers like conceptual analysis’, etc..
- (2) *Connectives*: ‘it is not the case that’, ‘and’, etc..
- (3) *Noun phrases*: ‘Bertrand Russell’, ‘Philosophers’, etc..
- (4) *Verb phrases*: ‘likes logic’, ‘like conceptual analysis’, etc..
- (5) Also: *nouns*, *verbs*, *pronouns*, etc., etc., etc..

Syntax: English vs. \mathcal{L}_1 .

English has *many* different sorts of expressions.

Some expressions of English

- (1) *Sentences*: ‘Bertrand Russell likes logic’, ‘Philosophers like conceptual analysis’, etc..
- (2) *Connectives*: ‘it is not the case that’, ‘and’, etc..
- (3) *Noun phrases*: ‘Bertrand Russell’, ‘Philosophers’, etc..
- (4) *Verb phrases*: ‘likes logic’, ‘like conceptual analysis’, etc..
- (5) *Also*: *nouns*, *verbs*, *pronouns*, etc., etc., etc..

\mathcal{L}_1 has *just two* sorts of basic expressions.

Some basic expressions of \mathcal{L}_1

Syntax: English vs. \mathcal{L}_1 .

English has *many* different sorts of expressions.

Some expressions of English

- (1) *Sentences*: ‘Bertrand Russell likes logic’, ‘Philosophers like conceptual analysis’, etc..
- (2) *Connectives*: ‘it is not the case that’, ‘and’, etc..
- (3) *Noun phrases*: ‘Bertrand Russell’, ‘Philosophers’, etc..
- (4) *Verb phrases*: ‘likes logic’, ‘like conceptual analysis’, etc..
- (5) *Also*: *nouns*, *verbs*, *pronouns*, etc., etc., etc..

\mathcal{L}_1 has *just two* sorts of basic expressions.

Some basic expressions of \mathcal{L}_1

- (1) *Sentence letters*: e.g. ‘P’, ‘Q’.

Syntax: English vs. \mathcal{L}_1 .

English has *many* different sorts of expressions.

Some expressions of English

- (1) *Sentences*: ‘Bertrand Russell likes logic’, ‘Philosophers like conceptual analysis’, etc..
- (2) *Connectives*: ‘it is not the case that’, ‘and’, etc..
- (3) *Noun phrases*: ‘Bertrand Russell’, ‘Philosophers’, etc..
- (4) *Verb phrases*: ‘likes logic’, ‘like conceptual analysis’, etc..
- (5) *Also*: *nouns*, *verbs*, *pronouns*, etc., etc., etc..

\mathcal{L}_1 has *just two* sorts of basic expressions.

Some basic expressions of \mathcal{L}_1

- (1) *Sentence letters*: e.g. ‘P’, ‘Q’.
- (2) *Connectives*: e.g. ‘ \neg ’, ‘ \wedge ’. There are also brackets: ‘(’ and ‘)’.

Combining sentences and connectives makes new sentences.

Combining sentences and connectives makes new sentences.

Some complex sentences

Combining sentences and connectives makes new sentences.

Some complex sentences

- 'It is not the case that' and 'Bertrand Russell likes logic' make:
'It is not the case that Bertrand Russell likes logic'.

Combining sentences and connectives makes new sentences.

Some complex sentences

- 'It is not the case that' and 'Bertrand Russell likes logic' make:
 'It is not the case that Bertrand Russell likes logic'.
- '¬' and 'P' make: ' $\neg P$ '.

Combining sentences and connectives makes new sentences.

Some complex sentences

- ‘It is not the case that’ and ‘Bertrand Russell likes logic’ make:
‘It is not the case that Bertrand Russell likes logic.’
- ‘ \neg ’ and ‘P’ make: ‘ $\neg P$ ’.
- ‘Bertrand Russell likes logic’ and ‘and’ and ‘Philosophers like conceptual analysis’ make:
‘Bertrand Russell likes logic and philosophers like conceptual analysis.’

Combining sentences and connectives makes new sentences.

Some complex sentences

- ‘It is not the case that’ and ‘Bertrand Russell likes logic’ make:
‘It is not the case that Bertrand Russell likes logic.’
- ‘ \neg ’ and ‘P’ make: ‘ $\neg P$ ’.
- ‘Bertrand Russell likes logic’ and ‘and’ and ‘Philosophers like conceptual analysis’ make:
‘Bertrand Russell likes logic and philosophers like conceptual analysis.’
- ‘P’, ‘ \wedge ’ and ‘Q’ make: ‘ $(P \wedge Q)$ ’.

Combining sentences and connectives makes new sentences.

Some complex sentences

- ‘It is not the case that’ and ‘Bertrand Russell likes logic’ make:
‘It is not the case that Bertrand Russell likes logic.’
- ‘ \neg ’ and ‘P’ make: ‘ $\neg P$ ’.
- ‘Bertrand Russell likes logic’ and ‘and’ and ‘Philosophers like conceptual analysis’ make:
‘Bertrand Russell likes logic and philosophers like conceptual analysis.’
- ‘P’, ‘ \wedge ’ and ‘Q’ make: ‘ $(P \wedge Q)$ ’.

Logic convention: no quotes around \mathcal{L}_1 -expressions.

Combining sentences and connectives makes new sentences.

Some complex sentences

- ‘It is not the case that’ and ‘Bertrand Russell likes logic’ make:
‘It is not the case that Bertrand Russell likes logic.’
- ‘ \neg ’ and ‘P’ make: ‘ $\neg P$ ’.
- ‘Bertrand Russell likes logic’ and ‘and’ and ‘Philosophers like conceptual analysis’ make:
‘Bertrand Russell likes logic and philosophers like conceptual analysis.’
- ‘P’, ‘ \wedge ’ and ‘Q’ make: ‘ $(P \wedge Q)$ ’.

Logic convention: no quotes around \mathcal{L}_1 -expressions.

- P, \wedge and Q make: $(P \wedge Q)$.

Connectives

Here's the full list of \mathcal{L}_1 -connectives.

name	in English	symbol
conjunction	and	\wedge
disjunction	or	\vee
negation	it is not the case that	\neg
arrow	if ... then	\rightarrow
double arrow	if and only if	\leftrightarrow

Connectives

Here's the full list of \mathcal{L}_1 -connectives.

name	in English	symbol
conjunction	and	\wedge
disjunction	or	\vee
negation	it is not the case that	\neg
arrow	if ... then	\rightarrow
double arrow	if and only if	\leftrightarrow

Connectives

Here's the full list of \mathcal{L}_1 -connectives.

name	in English	symbol
conjunction	and	\wedge
disjunction	or	\vee
negation	it is not the case that	\neg
arrow	if ... then	\rightarrow
double arrow	if and only if	\leftrightarrow

Connectives

Here's the full list of \mathcal{L}_1 -connectives.

name	in English	symbol
conjunction	and	\wedge
disjunction	or	\vee
negation	it is not the case that	\neg
arrow	if ... then	\rightarrow
double arrow	if and only if	\leftrightarrow

Connectives

Here's the full list of \mathcal{L}_1 -connectives.

name	in English	symbol
conjunction	and	\wedge
disjunction	or	\vee
negation	it is not the case that	\neg
arrow	if ... then	\rightarrow
double arrow	if and only if	\leftrightarrow

Connectives

Here's the full list of \mathcal{L}_1 -connectives.

name	in English	symbol
conjunction	and	\wedge
disjunction	or	\vee
negation	it is not the case that	\neg
arrow	if ... then	\rightarrow
double arrow	if and only if	\leftrightarrow

The syntax of \mathcal{L}_1

Here's the official definition of \mathcal{L}_1 -sentence.

The syntax of \mathcal{L}_1

Here's the official definition of \mathcal{L}_1 -sentence.

Definition

The syntax of \mathcal{L}_1

Here's the official definition of \mathcal{L}_1 -sentence.

Definition

- (i) All sentence letters are sentences of \mathcal{L}_1 :
- $P, Q, R, P_1, Q_1, R_1, P_2, Q_2, R_2, P_3, \dots$

The syntax of \mathcal{L}_1

Here's the official definition of \mathcal{L}_1 -sentence.

Definition

- (i) All sentence letters are sentences of \mathcal{L}_1 :
 - $P, Q, R, P_1, Q_1, R_1, P_2, Q_2, R_2, P_3, \dots$
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then so are:
 - $\neg\phi$

The syntax of \mathcal{L}_1

Here's the official definition of \mathcal{L}_1 -sentence.

Definition

- (i) All sentence letters are sentences of \mathcal{L}_1 :
 - $P, Q, R, P_1, Q_1, R_1, P_2, Q_2, R_2, P_3, \dots$
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then so are:
 - $\neg\phi$
 - $(\phi \wedge \psi)$

The syntax of \mathcal{L}_1

Here's the official definition of \mathcal{L}_1 -sentence.

Definition

- (i) All sentence letters are sentences of \mathcal{L}_1 :
 - $P, Q, R, P_1, Q_1, R_1, P_2, Q_2, R_2, P_3, \dots$
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then so are:
 - $\neg\phi$
 - $(\phi \wedge \psi)$
 - $(\phi \vee \psi)$

The syntax of \mathcal{L}_1

Here's the official definition of \mathcal{L}_1 -sentence.

Definition

- (i) All sentence letters are sentences of \mathcal{L}_1 :
 - $P, Q, R, P_1, Q_1, R_1, P_2, Q_2, R_2, P_3, \dots$
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then so are:
 - $\neg\phi$
 - $(\phi \wedge \psi)$
 - $(\phi \vee \psi)$
 - $(\phi \rightarrow \psi)$

The syntax of \mathcal{L}_1

Here's the official definition of \mathcal{L}_1 -sentence.

Definition

- (i) All sentence letters are sentences of \mathcal{L}_1 :
 - $P, Q, R, P_1, Q_1, R_1, P_2, Q_2, R_2, P_3, \dots$
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then so are:
 - $\neg\phi$
 - $(\phi \wedge \psi)$
 - $(\phi \vee \psi)$
 - $(\phi \rightarrow \psi)$
 - $(\phi \leftrightarrow \psi)$

The syntax of \mathcal{L}_1

Here's the official definition of \mathcal{L}_1 -sentence.

Definition

- (i) All sentence letters are sentences of \mathcal{L}_1 :
 - $P, Q, R, P_1, Q_1, R_1, P_2, Q_2, R_2, P_3, \dots$
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then so are:
 - $\neg\phi$
 - $(\phi \wedge \psi)$
 - $(\phi \vee \psi)$
 - $(\phi \rightarrow \psi)$
 - $(\phi \leftrightarrow \psi)$
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

The syntax of \mathcal{L}_1

Here's the official definition of \mathcal{L}_1 -sentence.

Definition

- (i) All sentence letters are sentences of \mathcal{L}_1 :
 - $P, Q, R, P_1, Q_1, R_1, P_2, Q_2, R_2, P_3, \dots$
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then so are:
 - $\neg\phi$
 - $(\phi \wedge \psi)$
 - $(\phi \vee \psi)$
 - $(\phi \rightarrow \psi)$
 - $(\phi \leftrightarrow \psi)$
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

Greek letters: ϕ ('PHI') and ψ ('PSI'): not part of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1

Example

The following is a sentence of \mathcal{L}_1 :

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1

Example

The following is a sentence of \mathcal{L}_1 :

P

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1

Example

The following is a sentence of \mathcal{L}_1 :

$$P \quad Q$$

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1

Example

The following is a sentence of \mathcal{L}_1 :

$$(P \wedge Q)$$

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1

Example

The following is a sentence of \mathcal{L}_1 :

$$(P \wedge Q) \quad R_{45}$$

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1

Example

The following is a sentence of \mathcal{L}_1 :

$$(P \wedge Q) \quad \neg R_{45}$$

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1 **Example**

The following is a sentence of \mathcal{L}_1 :

$$(P \wedge Q) \quad P \quad \neg R_{45}$$

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1

Example

The following is a sentence of \mathcal{L}_1 :

$$(P \wedge Q) \quad (P \vee \neg R_{45})$$

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1

Example

The following is a sentence of \mathcal{L}_1 :

$$((P \wedge Q) \rightarrow (P \vee \neg R_{45}))$$

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1

Example

The following is a sentence of \mathcal{L}_1 :

$$((P \wedge Q) \rightarrow (P \vee \neg R_{45})) \quad P_3$$

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1

Example

The following is a sentence of \mathcal{L}_1 :

$$((P \wedge Q) \rightarrow (P \vee \neg R_{45})) \quad P_3 \quad R$$

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1 **Example**

The following is a sentence of \mathcal{L}_1 :

$$((P \wedge Q) \rightarrow (P \vee \neg R_{45})) \quad (P_3 \vee R)$$

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1

Example

The following is a sentence of \mathcal{L}_1 :

$$((P \wedge Q) \rightarrow (P \vee \neg R_{45})) \quad (P_3 \vee R) \quad R$$

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1

Example

The following is a sentence of \mathcal{L}_1 :

$$((P \wedge Q) \rightarrow (P \vee \neg R_{45})) \quad ((P_3 \vee R) \vee R)$$

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1

Example

The following is a sentence of \mathcal{L}_1 :

$$((P \wedge Q) \rightarrow (P \vee \neg R_{45})) \quad \neg((P_3 \vee R) \vee R)$$

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1

Example

The following is a sentence of \mathcal{L}_1 :

$$(((P \wedge Q) \rightarrow (P \vee \neg R_{45})) \leftrightarrow \neg((P_3 \vee R) \vee R))$$

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1

Example

The following is a sentence of \mathcal{L}_1 :

$$\neg(((P \wedge Q) \rightarrow (P \vee \neg R_{45})) \leftrightarrow \neg((P_3 \vee R) \vee R))$$

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

How to build a sentence of \mathcal{L}_1

Example

The following is a sentence of \mathcal{L}_1 :

$$\neg\neg(((P \wedge Q) \rightarrow (P \vee \neg R_{45})) \leftrightarrow \neg((P_3 \vee R) \vee R))$$

Definition of \mathcal{L}_1 -sentences (repeated from previous page)

- (i) All sentence letters are sentences of \mathcal{L}_1 .
- (ii) If ϕ and ψ are sentences of \mathcal{L}_1 , then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of \mathcal{L}_1 .
- (iii) Nothing else is a sentence of \mathcal{L}_1 .

I mentioned that ϕ and ψ are *not* part of \mathcal{L}_1 .

I mentioned that ϕ and ψ are *not* part of \mathcal{L}_1 .

- $\neg P$ is a \mathcal{L}_1 -sentence.

I mentioned that ϕ and ψ are *not* part of \mathcal{L}_1 .

- $\neg P$ is a \mathcal{L}_1 -sentence.
- $\neg\phi$ describes many \mathcal{L}_1 -sentences

I mentioned that ϕ and ψ are *not* part of \mathcal{L}_1 .

- $\neg P$ is a \mathcal{L}_1 -sentence.
- $\neg\phi$ describes many \mathcal{L}_1 -sentences
e.g. $\neg P$, $\neg(Q \vee R)$, $\neg(P \leftrightarrow (Q \vee R))$, ...

I mentioned that ϕ and ψ are *not* part of \mathcal{L}_1 .

- $\neg P$ is a \mathcal{L}_1 -sentence.
- $\neg\phi$ describes many \mathcal{L}_1 -sentences (**but is not one itself**).
e.g. $\neg P$, $\neg(Q \vee R)$, $\neg(P \leftrightarrow (Q \vee R))$, ...

I mentioned that ϕ and ψ are *not* part of \mathcal{L}_1 .

- $\neg P$ is a \mathcal{L}_1 -sentence.
- $\neg\phi$ describes many \mathcal{L}_1 -sentences (but is not one itself).
e.g. $\neg P$, $\neg(Q \vee R)$, $\neg(P \leftrightarrow (Q \vee R))$, ...

ϕ and ψ are part of the metalanguage, not the object one.

Object language

The object language is the one we are theorising *about*.

- The object language is \mathcal{L}_1 .

Metalanguage

The metalanguage is the one we are theorising *in*.

- The metalanguage is (augmented) English.

ϕ and ψ are used as variables in the metalanguage:
in order to generalise about sentences of the object language.

Bracketing conventions

Bracketing conventions

There are conventions for dropping brackets in \mathcal{L}_1 .

Some are similar to rules used for $+$ and \times in arithmetic.

Bracketing conventions

There are conventions for dropping brackets in \mathcal{L}_1 .
Some are similar to rules used for $+$ and \times in arithmetic.

Example in arithmetic

Bracketing conventions

There are conventions for dropping brackets in \mathcal{L}_1 .
Some are similar to rules used for $+$ and \times in arithmetic.

Example in arithmetic

- $4 + 5 \times 3$

Bracketing conventions

There are conventions for dropping brackets in \mathcal{L}_1 .
Some are similar to rules used for + and \times in arithmetic.

Example in arithmetic

- $4 + 5 \times 3$ does **not** abbreviate $(4 + 5) \times 3$.

Bracketing conventions

There are conventions for dropping brackets in \mathcal{L}_1 .

Some are similar to rules used for $+$ and \times in arithmetic.

Example in arithmetic

- $4 + 5 \times 3$ does **not** abbreviate $(4 + 5) \times 3$.
- \times 'binds more strongly' than $+$.

Bracketing conventions

There are conventions for dropping brackets in \mathcal{L}_1 .

Some are similar to rules used for + and \times in arithmetic.

Example in arithmetic

- $4 + 5 \times 3$ does **not** abbreviate $(4 + 5) \times 3$.
- \times 'binds more strongly' than +.
 $4 + 5 \times 3$ abbreviates $4 + (5 \times 3)$.

Bracketing conventions

There are conventions for dropping brackets in \mathcal{L}_1 .
Some are similar to rules used for + and \times in arithmetic.

Example in arithmetic

- $4 + 5 \times 3$ does **not** abbreviate $(4 + 5) \times 3$.
- \times 'binds more strongly' than +.
 $4 + 5 \times 3$ abbreviates $4 + (5 \times 3)$.

Conventions in \mathcal{L}_1

Bracketing conventions

There are conventions for dropping brackets in \mathcal{L}_1 .

Some are similar to rules used for $+$ and \times in arithmetic.

Example in arithmetic

- $4 + 5 \times 3$ does **not** abbreviate $(4 + 5) \times 3$.
- \times 'binds more strongly' than $+$.
 $4 + 5 \times 3$ abbreviates $4 + (5 \times 3)$.

Conventions in \mathcal{L}_1

- \wedge and \vee bind more strongly than \rightarrow and \leftrightarrow .

Bracketing conventions

There are conventions for dropping brackets in \mathcal{L}_1 .
Some are similar to rules used for $+$ and \times in arithmetic.

Example in arithmetic

- $4 + 5 \times 3$ does **not** abbreviate $(4 + 5) \times 3$.
- \times 'binds more strongly' than $+$.
 $4 + 5 \times 3$ abbreviates $4 + (5 \times 3)$.

Conventions in \mathcal{L}_1

- \wedge and \vee bind more strongly than \rightarrow and \leftrightarrow .
 $(P \rightarrow Q \wedge R)$ abbreviates $(P \rightarrow (Q \wedge R))$.

Bracketing conventions

There are conventions for dropping brackets in \mathcal{L}_1 .
Some are similar to rules used for $+$ and \times in arithmetic.

Example in arithmetic

- $4 + 5 \times 3$ does **not** abbreviate $(4 + 5) \times 3$.
- \times 'binds more strongly' than $+$.
 $4 + 5 \times 3$ abbreviates $4 + (5 \times 3)$.

Conventions in \mathcal{L}_1

- \wedge and \vee bind more strongly than \rightarrow and \leftrightarrow .
 $(P \rightarrow Q \wedge R)$ abbreviates $(P \rightarrow (Q \wedge R))$.
- One may drop outer brackets.

Bracketing conventions

There are conventions for dropping brackets in \mathcal{L}_1 .
Some are similar to rules used for $+$ and \times in arithmetic.

Example in arithmetic

- $4 + 5 \times 3$ does **not** abbreviate $(4 + 5) \times 3$.
- \times 'binds more strongly' than $+$.
 $4 + 5 \times 3$ abbreviates $4 + (5 \times 3)$.

Conventions in \mathcal{L}_1

- \wedge and \vee bind more strongly than \rightarrow and \leftrightarrow .
 $(P \rightarrow Q \wedge R)$ abbreviates $(P \rightarrow (Q \wedge R))$.
- One may drop outer brackets.
 $P \wedge (Q \rightarrow \neg P_4)$ abbreviates $(P \wedge (Q \rightarrow \neg P_4))$.

Bracketing conventions

There are conventions for dropping brackets in \mathcal{L}_1 .
Some are similar to rules used for $+$ and \times in arithmetic.

Example in arithmetic

- $4 + 5 \times 3$ does **not** abbreviate $(4 + 5) \times 3$.
- \times 'binds more strongly' than $+$.
 $4 + 5 \times 3$ abbreviates $4 + (5 \times 3)$.

Conventions in \mathcal{L}_1

- \wedge and \vee bind more strongly than \rightarrow and \leftrightarrow .
 $(P \rightarrow Q \wedge R)$ abbreviates $(P \rightarrow (Q \wedge R))$.
- One may drop outer brackets.
 $P \wedge (Q \rightarrow \neg P_4)$ abbreviates $(P \wedge (Q \rightarrow \neg P_4))$.
- One may drop brackets on strings of \wedge s or \vee s that are bracketed to the left.

Bracketing conventions

There are conventions for dropping brackets in \mathcal{L}_1 .
Some are similar to rules used for $+$ and \times in arithmetic.

Example in arithmetic

- $4 + 5 \times 3$ does **not** abbreviate $(4 + 5) \times 3$.
- \times 'binds more strongly' than $+$.
 $4 + 5 \times 3$ abbreviates $4 + (5 \times 3)$.

Conventions in \mathcal{L}_1

- \wedge and \vee bind more strongly than \rightarrow and \leftrightarrow .
 $(P \rightarrow Q \wedge R)$ abbreviates $(P \rightarrow (Q \wedge R))$.
- One may drop outer brackets.
 $P \wedge (Q \rightarrow \neg P_4)$ abbreviates $(P \wedge (Q \rightarrow \neg P_4))$.
- One may drop brackets on strings of \wedge s or \vee s that are bracketed to the left.
 $(P \wedge Q \wedge R)$ abbreviates $((P \wedge Q) \wedge R)$.

Bracketing conventions

There are conventions for dropping brackets in \mathcal{L}_1 .
Some are similar to rules used for $+$ and \times in arithmetic.

Example in arithmetic

- $4 + 5 \times 3$ does **not** abbreviate $(4 + 5) \times 3$.
- \times 'binds more strongly' than $+$.
 $4 + 5 \times 3$ abbreviates $4 + (5 \times 3)$.

Conventions in \mathcal{L}_1

- \wedge and \vee bind more strongly than \rightarrow and \leftrightarrow .
 $(P \rightarrow Q \wedge R)$ abbreviates $(P \rightarrow (Q \wedge R))$.
- One may drop outer brackets.
 $P \wedge (Q \rightarrow \neg P_4)$ abbreviates $(P \wedge (Q \rightarrow \neg P_4))$.
- One may drop brackets on strings of \wedge s or \vee s that are bracketed to the left.
 $(P \wedge Q \wedge R)$ abbreviates $((P \wedge Q) \wedge R)$.

Bracketing conventions

There are conventions for dropping brackets in \mathcal{L}_1 .
Some are similar to rules used for $+$ and \times in arithmetic.

Example in arithmetic

- $4 + 5 \times 3$ does **not** abbreviate $(4 + 5) \times 3$.
- \times 'binds more strongly' than $+$.
 $4 + 5 \times 3$ abbreviates $4 + (5 \times 3)$.

Conventions in \mathcal{L}_1

- \wedge and \vee bind more strongly than \rightarrow and \leftrightarrow .
 $(P \rightarrow Q \wedge R)$ abbreviates $(P \rightarrow (Q \wedge R))$.
- One may drop outer brackets.
 $P \wedge (Q \rightarrow \neg P_4)$ abbreviates $(P \wedge (Q \rightarrow \neg P_4))$.
- One may drop brackets on strings of \wedge s or \vee s that are bracketed to the left.
 $P \wedge Q \wedge R$ abbreviates $((P \wedge Q) \wedge R)$.

Semantics

Recall the characterisation of validity from week 1.

Semantics

Recall the characterisation of validity from week 1.

Characterisation

An argument is *logically valid* if and only if there is *no* interpretation of subject-specific expressions under which:

- (i) the premisses are all true, and
- (ii) the conclusion is false.

Semantics

Recall the characterisation of validity from week 1.

Characterisation

An argument is *logically valid* if and only if there is *no* interpretation of subject-specific expressions under which:

- (i) the premisses are all true, and
- (ii) the conclusion is false.

We'll adapt this characterisation to \mathcal{L}_1 .

Semantics

Recall the characterisation of validity from week 1.

Characterisation

An argument is *logically valid* if and only if there is *no* interpretation of subject-specific expressions under which:

- (i) the premisses are all true, and
- (ii) the conclusion is false.

We'll adapt this characterisation to \mathcal{L}_1 .

- Logical expressions: \neg , \wedge , \vee , \rightarrow and \leftrightarrow .

Semantics

Recall the characterisation of validity from week 1.

Characterisation

An argument is *logically valid* if and only if there is *no* interpretation of subject-specific expressions under which:

- (i) the premisses are all true, and
- (ii) the conclusion is false.

We'll adapt this characterisation to \mathcal{L}_1 .

- Logical expressions: $\neg, \wedge, \vee, \rightarrow$ and \leftrightarrow .
- Subject-specific expressions: P, Q, R, \dots

Semantics

Recall the characterisation of validity from week 1.

Characterisation

An argument is *logically valid* if and only if there is *no* interpretation of subject-specific expressions under which:

- (i) the premisses are all true, and
- (ii) the conclusion is false.

We'll adapt this characterisation to \mathcal{L}_1 .

- Logical expressions: $\neg, \wedge, \vee, \rightarrow$ and \leftrightarrow .
- Subject-specific expressions: P, Q, R, \dots
- Interpretation: \mathcal{L}_1 -structure.

\mathcal{L}_1 -structures

\mathcal{L}_1 -structures

We interpret sentence letters by assigning them truth-values:

\mathcal{L}_1 -structures

We interpret sentence letters by assigning them truth-values:
either T for True or F for False.

\mathcal{L}_1 -structures

We interpret sentence letters by assigning them truth-values: either T for True or F for False.

Definition

An \mathcal{L}_1 -structure is an assignment of exactly one truth-value (T or F) to every sentence letter of \mathcal{L}_1 .

\mathcal{L}_1 -structures

We interpret sentence letters by assigning them truth-values: either T for True or F for False.

Definition

An \mathcal{L}_1 -structure is an assignment of exactly one truth-value (T or F) to every sentence letter of \mathcal{L}_1 .

Examples

We can think of an \mathcal{L}_1 -structure as an infinite list that provides a value T or F for every sentence letter.

	P	Q	R	P_1	Q_1	R_1	P_2	Q_2	R_2	\dots
$\mathcal{A}:$	T	F	F	F	T	F	T	T	F	\dots

\mathcal{L}_1 -structures

We interpret sentence letters by assigning them truth-values: either T for True or F for False.

Definition

An \mathcal{L}_1 -structure is an assignment of exactly one truth-value (T or F) to every sentence letter of \mathcal{L}_1 .

Examples

We can think of an \mathcal{L}_1 -structure as an infinite list that provides a value T or F for every sentence letter.

	P	Q	R	P_1	Q_1	R_1	P_2	Q_2	R_2	\dots
$\mathcal{B}:$	F	F	F	F	F	F	F	F	F	\dots

We use \mathcal{A} , \mathcal{B} , etc. to stand for \mathcal{L}_1 -structures.

Truth-values of complex sentences 1/3

\mathcal{L}_1 -structures *only* directly specify truth-values for P, Q, R, \dots

Truth-values of complex sentences 1/3

\mathcal{L}_1 -structures *only* directly specify truth-values for P, Q, R, \dots

- The logical connectives have fixed meanings.

Truth-values of complex sentences 1/3

\mathcal{L}_1 -structures *only* directly specify truth-values for P, Q, R, \dots

- The logical connectives have fixed meanings.
- These determine the truth-values of complex sentences.

Truth-values of complex sentences 1/3

\mathcal{L}_1 -structures *only* directly specify truth-values for P, Q, R, \dots

- The logical connectives have fixed meanings.
- These determine the truth-values of complex sentences.
- Notation: $|\phi|_{\mathcal{A}}$ is the truth-value of ϕ under \mathcal{A} .

Truth-values of complex sentences 1/3

\mathcal{L}_1 -structures *only* directly specify truth-values for P, Q, R, \dots

- The logical connectives have fixed meanings.
- These determine the truth-values of complex sentences.
- Notation: $|\phi|_{\mathcal{A}}$ is the truth-value of ϕ under \mathcal{A} .
- For all \mathcal{L}_1 -structures \mathcal{A} and sentences ϕ we have either $|\phi|_{\mathcal{A}} = T$ or $|\phi|_{\mathcal{A}} = F$.

Truth-values of complex sentences 1/3

\mathcal{L}_1 -structures *only* directly specify truth-values for P, Q, R, \dots

- The logical connectives have fixed meanings.
- These determine the truth-values of complex sentences.
- Notation: $|\phi|_{\mathcal{A}}$ is the truth-value of ϕ under \mathcal{A} .
- For all \mathcal{L}_1 -structures \mathcal{A} and sentences ϕ we have either $|\phi|_{\mathcal{A}} = T$ or $|\phi|_{\mathcal{A}} = F$.

Truth-conditions for \neg

The meaning of \neg is summarised in its *truth table*.

ϕ		$\neg\phi$
T		F
F		T

Truth-values of complex sentences 1/3

\mathcal{L}_1 -structures *only* directly specify truth-values for P, Q, R, \dots

- The logical connectives have fixed meanings.
- These determine the truth-values of complex sentences.
- Notation: $|\phi|_{\mathcal{A}}$ is the truth-value of ϕ under \mathcal{A} .
- For all \mathcal{L}_1 -structures \mathcal{A} and sentences ϕ we have either $|\phi|_{\mathcal{A}} = T$ or $|\phi|_{\mathcal{A}} = F$.

Truth-conditions for \neg

The meaning of \neg is summarised in its *truth table*.

ϕ		$\neg\phi$
T		F
F		T

In words: $|\neg\phi|_{\mathcal{A}} = T$ if and only if $|\phi|_{\mathcal{A}} = F$.

Worked example 1

$|\phi|_{\mathcal{A}}$ is the truth-value of ϕ under \mathcal{A} .

ϕ	$\neg\phi$
T	F
F	T

Compute the following truth-values.

Let the structure \mathcal{A} be partially specified as follows.

P	Q	R	P_1	Q_1	R_1	P_2	Q_2	R_2	\dots
T	F	F	F	T	F	T	T	F	\dots

Compute:

$$\begin{array}{lll}
 |P|_{\mathcal{A}} = & |Q|_{\mathcal{A}} = & |R_1|_{\mathcal{A}} = \\
 |\neg P|_{\mathcal{A}} = & |\neg Q|_{\mathcal{A}} = & |\neg R_1|_{\mathcal{A}} = \\
 |\neg\neg P|_{\mathcal{A}} = & |\neg\neg Q|_{\mathcal{A}} = & |\neg\neg R_1|_{\mathcal{A}} =
 \end{array}$$

Worked example 1

$|\phi|_{\mathcal{A}}$ is the truth-value of ϕ under \mathcal{A} .

ϕ	$\neg\phi$
T	F
F	T

Compute the following truth-values.

Let the structure \mathcal{A} be partially specified as follows.

<i>P</i>	Q	R	<i>P</i> ₁	<i>Q</i> ₁	<i>R</i> ₁	<i>P</i> ₂	<i>Q</i> ₂	<i>R</i> ₂	...
T	F	F	F	T	F	T	T	F	...

Compute:

$$\begin{array}{lll}
 |P|_{\mathcal{A}} = \mathbf{T} & |Q|_{\mathcal{A}} = & |R_1|_{\mathcal{A}} = \\
 |\neg P|_{\mathcal{A}} = & |\neg Q|_{\mathcal{A}} = & |\neg R_1|_{\mathcal{A}} = \\
 |\neg\neg P|_{\mathcal{A}} = & |\neg\neg Q|_{\mathcal{A}} = & |\neg\neg R_1|_{\mathcal{A}} =
 \end{array}$$

Worked example 1

$|\phi|_{\mathcal{A}}$ is the truth-value of ϕ under \mathcal{A} .

ϕ	$\neg\phi$
T	F
F	T

Compute the following truth-values.

Let the structure \mathcal{A} be partially specified as follows.

P	Q	R	P_1	Q_1	R_1	P_2	Q_2	R_2	\dots
T	F	F	F	T	F	T	T	F	\dots

Compute:

$ P _{\mathcal{A}} = \text{T}$	$ Q _{\mathcal{A}} = \text{F}$	$ R_1 _{\mathcal{A}} =$
$ \neg P _{\mathcal{A}} =$	$ \neg Q _{\mathcal{A}} =$	$ \neg R_1 _{\mathcal{A}} =$
$ \neg\neg P _{\mathcal{A}} =$	$ \neg\neg Q _{\mathcal{A}} =$	$ \neg\neg R_1 _{\mathcal{A}} =$

Worked example 1

$|\phi|_{\mathcal{A}}$ is the truth-value of ϕ under \mathcal{A} .

ϕ	$\neg\phi$
T	F
F	T

Compute the following truth-values.

Let the structure \mathcal{A} be partially specified as follows.

P	Q	R	P_1	Q_1	R_1	P_2	Q_2	R_2	\dots
T	F	F	F	T	F	T	T	F	\dots

Compute:

$ P _{\mathcal{A}} = \text{T}$	$ Q _{\mathcal{A}} = \text{F}$	$ R_1 _{\mathcal{A}} = \text{F}$
$ \neg P _{\mathcal{A}} =$	$ \neg Q _{\mathcal{A}} =$	$ \neg R_1 _{\mathcal{A}} =$
$ \neg\neg P _{\mathcal{A}} =$	$ \neg\neg Q _{\mathcal{A}} =$	$ \neg\neg R_1 _{\mathcal{A}} =$

Worked example 1

$|\phi|_{\mathcal{A}}$ is the truth-value of ϕ under \mathcal{A} .

ϕ	$\neg\phi$
T	F
F	T

Compute the following truth-values.

Let the structure \mathcal{A} be partially specified as follows.

P	Q	R	P_1	Q_1	R_1	P_2	Q_2	R_2	\dots
T	F	F	F	T	F	T	T	F	\dots

Compute:

$ P _{\mathcal{A}} = \text{T}$	$ Q _{\mathcal{A}} = \text{F}$	$ R_1 _{\mathcal{A}} = \text{F}$
$ \neg P _{\mathcal{A}} = \text{F}$	$ \neg Q _{\mathcal{A}} =$	$ \neg R_1 _{\mathcal{A}} =$
$ \neg\neg P _{\mathcal{A}} =$	$ \neg\neg Q _{\mathcal{A}} =$	$ \neg\neg R_1 _{\mathcal{A}} =$

Worked example 1

$|\phi|_{\mathcal{A}}$ is the truth-value of ϕ under \mathcal{A} .

ϕ	$\neg\phi$
T	F
F	T

Compute the following truth-values.

Let the structure \mathcal{A} be partially specified as follows.

P	Q	R	P_1	Q_1	R_1	P_2	Q_2	R_2	\dots
T	F	F	F	T	F	T	T	F	\dots

Compute:

$ P _{\mathcal{A}} = \text{T}$	$ Q _{\mathcal{A}} = \text{F}$	$ R_1 _{\mathcal{A}} = \text{F}$
$ \neg P _{\mathcal{A}} = \text{F}$	$ \neg Q _{\mathcal{A}} = \text{T}$	$ \neg R_1 _{\mathcal{A}} =$
$ \neg\neg P _{\mathcal{A}} =$	$ \neg\neg Q _{\mathcal{A}} =$	$ \neg\neg R_1 _{\mathcal{A}} =$

Worked example 1

$|\phi|_{\mathcal{A}}$ is the truth-value of ϕ under \mathcal{A} .

ϕ	$\neg\phi$
T	F
F	T

Compute the following truth-values.

Let the structure \mathcal{A} be partially specified as follows.

P	Q	R	P_1	Q_1	R_1	P_2	Q_2	R_2	\dots
T	F	F	F	T	F	T	T	F	\dots

Compute:

$ P _{\mathcal{A}} = \text{T}$	$ Q _{\mathcal{A}} = \text{F}$	$ R_1 _{\mathcal{A}} = \text{F}$
$ \neg P _{\mathcal{A}} = \text{F}$	$ \neg Q _{\mathcal{A}} = \text{T}$	$ \neg R_1 _{\mathcal{A}} = \text{T}$
$ \neg\neg P _{\mathcal{A}} =$	$ \neg\neg Q _{\mathcal{A}} =$	$ \neg\neg R_1 _{\mathcal{A}} =$

Worked example 1

$|\phi|_{\mathcal{A}}$ is the truth-value of ϕ under \mathcal{A} .

ϕ	$\neg\phi$
T	F
F	T

Compute the following truth-values.

Let the structure \mathcal{A} be partially specified as follows.

P	Q	R	P_1	Q_1	R_1	P_2	Q_2	R_2	\dots
T	F	F	F	T	F	T	T	F	\dots

Compute:

$ P _{\mathcal{A}} = \text{T}$	$ Q _{\mathcal{A}} = \text{F}$	$ R_1 _{\mathcal{A}} = \text{F}$
$ \neg P _{\mathcal{A}} = \text{F}$	$ \neg Q _{\mathcal{A}} = \text{T}$	$ \neg R_1 _{\mathcal{A}} = \text{T}$
$ \neg\neg P _{\mathcal{A}} = \text{T}$	$ \neg\neg Q _{\mathcal{A}} =$	$ \neg\neg R_1 _{\mathcal{A}} =$

Worked example 1

$|\phi|_{\mathcal{A}}$ is the truth-value of ϕ under \mathcal{A} .

ϕ	$\neg\phi$
T	F
F	T

Compute the following truth-values.

Let the structure \mathcal{A} be partially specified as follows.

P	Q	R	P_1	Q_1	R_1	P_2	Q_2	R_2	\dots
T	F	F	F	T	F	T	T	F	\dots

Compute:

$ P _{\mathcal{A}} = \text{T}$	$ Q _{\mathcal{A}} = \text{F}$	$ R_1 _{\mathcal{A}} = \text{F}$
$ \neg P _{\mathcal{A}} = \text{F}$	$ \neg Q _{\mathcal{A}} = \text{T}$	$ \neg R_1 _{\mathcal{A}} = \text{T}$
$ \neg\neg P _{\mathcal{A}} = \text{T}$	$ \neg\neg Q _{\mathcal{A}} = \text{F}$	$ \neg\neg R_1 _{\mathcal{A}} =$

Worked example 1

$|\phi|_{\mathcal{A}}$ is the truth-value of ϕ under \mathcal{A} .

ϕ	$\neg\phi$
T	F
F	T

Compute the following truth-values.

Let the structure \mathcal{A} be partially specified as follows.

P	Q	R	P_1	Q_1	R_1	P_2	Q_2	R_2	\dots
T	F	F	F	T	F	T	T	F	\dots

Compute:

$ P _{\mathcal{A}} = \text{T}$	$ Q _{\mathcal{A}} = \text{F}$	$ R_1 _{\mathcal{A}} = \text{F}$
$ \neg P _{\mathcal{A}} = \text{F}$	$ \neg Q _{\mathcal{A}} = \text{T}$	$ \neg R_1 _{\mathcal{A}} = \text{T}$
$ \neg\neg P _{\mathcal{A}} = \text{T}$	$ \neg\neg Q _{\mathcal{A}} = \text{F}$	$ \neg\neg R_1 _{\mathcal{A}} = \text{F}$

Truth-values of complex sentences 2/3

Truth-conditions for \wedge and \vee

The meanings of \wedge and \vee are given by the truth tables:

ϕ	ψ	$(\phi \wedge \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

ϕ	ψ	$(\phi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

Truth-values of complex sentences 2/3

Truth-conditions for \wedge and \vee

The meanings of \wedge and \vee are given by the truth tables:

ϕ	ψ	$(\phi \wedge \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

ϕ	ψ	$(\phi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

Truth-values of complex sentences 2/3

Truth-conditions for \wedge and \vee

The meanings of \wedge and \vee are given by the truth tables:

ϕ	ψ	$(\phi \wedge \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

ϕ	ψ	$(\phi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

$|(\phi \wedge \psi)|_{\mathcal{A}} = \text{T}$ if and only if $|\phi|_{\mathcal{A}} = \text{T}$ and $|\psi|_{\mathcal{A}} = \text{T}$.

Truth-values of complex sentences 2/3

Truth-conditions for \wedge and \vee

The meanings of \wedge and \vee are given by the truth tables:

ϕ	ψ	$(\phi \wedge \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

ϕ	ψ	$(\phi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

$|(\phi \wedge \psi)|_{\mathcal{A}} = \text{T}$ if and only if $|\phi|_{\mathcal{A}} = \text{T}$ and $|\psi|_{\mathcal{A}} = \text{T}$.

Truth-values of complex sentences 2/3

Truth-conditions for \wedge and \vee

The meanings of \wedge and \vee are given by the truth tables:

ϕ	ψ	$(\phi \wedge \psi)$	ϕ	ψ	$(\phi \vee \psi)$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	F	F	T	T
F	F	F	F	F	F

$|(\phi \wedge \psi)|_{\mathcal{A}} = \text{T}$ if and only if $|\phi|_{\mathcal{A}} = \text{T}$ and $|\psi|_{\mathcal{A}} = \text{T}$.

$|(\phi \vee \psi)|_{\mathcal{A}} = \text{T}$ if and only if $|\phi|_{\mathcal{A}} = \text{T}$ or $|\psi|_{\mathcal{A}} = \text{T}$ (or both).

*Truth-values of complex sentences 3/3***Truth-conditions for \rightarrow and \leftrightarrow**

The meanings of \rightarrow and \leftrightarrow are given by the truth tables:

ϕ	ψ	$(\phi \rightarrow \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Truth-values of complex sentences 3/3

Truth-conditions for \rightarrow and \leftrightarrow

The meanings of \rightarrow and \leftrightarrow are given by the truth tables:

ϕ	ψ	$(\phi \rightarrow \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Truth-values of complex sentences 3/3

Truth-conditions for \rightarrow and \leftrightarrow

The meanings of \rightarrow and \leftrightarrow are given by the truth tables:

ϕ	ψ	$(\phi \rightarrow \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

$|(\phi \rightarrow \psi)|_{\mathcal{A}} = \text{T}$ if and only if $|\phi|_{\mathcal{A}} = \text{F}$ or $|\psi|_{\mathcal{A}} = \text{T}$.

Truth-values of complex sentences 3/3

Truth-conditions for \rightarrow and \leftrightarrow

The meanings of \rightarrow and \leftrightarrow are given by the truth tables:

ϕ	ψ	$(\phi \rightarrow \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

$|(\phi \rightarrow \psi)|_{\mathcal{A}} = \text{T}$ if and only if $|\phi|_{\mathcal{A}} = \text{F}$ or $|\psi|_{\mathcal{A}} = \text{T}$.

Truth-values of complex sentences 3/3

Truth-conditions for \rightarrow and \leftrightarrow

The meanings of \rightarrow and \leftrightarrow are given by the truth tables:

ϕ	ψ	$(\phi \rightarrow \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

$|(\phi \rightarrow \psi)|_{\mathcal{A}} = \text{T}$ if and only if $|\phi|_{\mathcal{A}} = \text{F}$ or $|\psi|_{\mathcal{A}} = \text{T}$.

$|(\phi \leftrightarrow \psi)|_{\mathcal{A}} = \text{T}$ if and only if $|\phi|_{\mathcal{A}} = |\psi|_{\mathcal{A}}$.

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

① $|(P \rightarrow Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

① $|(P \rightarrow Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

① $|(P \rightarrow Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

① $|(P \rightarrow Q)|_{\mathcal{B}} = \text{F}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

① $|(P \rightarrow Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

① $|(P \rightarrow Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

① $|(P \rightarrow Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

① $|(P \rightarrow Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \text{F}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

- ① $|(P \rightarrow Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \text{F}$
- ② $|\neg(P \rightarrow Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

- ① $|(P \rightarrow Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \text{F}$
- ② $|\neg(P \rightarrow Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

- ① $|(P \rightarrow Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \text{F}$
- ② $|\neg(P \rightarrow Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

- ① $|(P \rightarrow Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \text{F}$
- ② $|\neg(P \rightarrow Q)|_{\mathcal{B}} = \text{T}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

- ① $|(P \rightarrow Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \text{F}$
- ② $|\neg(P \rightarrow Q)|_{\mathcal{B}} = \text{T}$
- ③ $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

- ① $|(P \rightarrow Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \text{F}$
- ② $|\neg(P \rightarrow Q)|_{\mathcal{B}} = \text{T}$
- ③ $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

- ① $|(P \rightarrow Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \text{F}$
- ② $|\neg(P \rightarrow Q)|_{\mathcal{B}} = \text{T}$
- ③ $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

- ① $|(P \rightarrow Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \text{F}$
- ② $|\neg(P \rightarrow Q)|_{\mathcal{B}} = \text{T}$
- ③ $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}} = \text{F}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$ under \mathcal{B} ?

- ① $|(P \rightarrow Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \text{F}$
- ② $|\neg(P \rightarrow Q)|_{\mathcal{B}} = \text{T}$
- ③ $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}} = \text{F}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

P	Q	\neg	$(P \rightarrow Q)$	\rightarrow	$(P \wedge Q)$
T	F	T	F	T	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg(P \rightarrow Q)$	\rightarrow	$(P \wedge Q)$
T	F	T	F	T F F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

P	Q	\neg	$(P \rightarrow Q)$	\rightarrow	$(P \wedge Q)$
T	F	T	F	T	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

P	Q	\neg	$(P \rightarrow Q)$	\rightarrow	$(P \wedge Q)$
T	F	T	F	F	T F F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

P	Q	\neg	$(P \rightarrow Q)$	\rightarrow	$(P \wedge Q)$
T	F	T	F	F	T F F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
F	T	T	F	F	F
		F	T	F	T
		F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$						
T	F	T	T	F	F	T	F	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \rightarrow Q) \rightarrow (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$							
T	F	T	T	F	F	F	T	F	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$
T	T	
T	F	
F	T	
F	F	

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$
T	T	T
T	F	
F	T	
F	F	

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$
T	T	T
T	F	T
F	T	
F	F	

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$
T	T	T
T	F	T
F	T	F
F	F	

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$
T	T	T
T	F	T
F	T	F
F	F	F

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg (P \rightarrow Q) \rightarrow (P \wedge Q)$
T	T	T
T	F	T
F	T	F
F	F	F

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$	
T	T	T	T
T	F	T	T
F	T	F	
F	F	F	

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$	
T	T	T	T
T	F	T	T
F	T	F	F
F	F	F	

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$	
T	T	T	T
T	F	T	T
F	T	F	F
F	F	F	F

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$		
T	T	T	T	T
T	F	T		T
F	T	F		F
F	F	F		F

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$		
T	T	T	T	T
T	F	T	F	T
F	T	F		F
F	F	F		F

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$		
T	T	T	T	T
T	F	T	F	T
F	T	F	T	F
F	F	F		F

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$		
T	T	T	T	T
T	F	T	F	T
F	T	F	T	F
F	F	F	F	F

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$			
T	T	T	T	T	T
T	F	T	F	T	
F	T	F	T	F	
F	F	F	F	F	

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$			
T	T	T	T	T	T
T	F	T	F	T	F
F	T	F	T	F	
F	F	F	F	F	

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$			
T	T	T	T	T	T
T	F	T	F	T	F
F	T	F	T	F	T
F	F	F	F	F	

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$			
T	T	T	T	T	T
T	F	T	F	T	F
F	T	F	T	F	T
F	F	F	F	F	F

Using the same technique we can fill in the full truth table for

$$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$				
T	T	T	T	T	T	T
T	F	T		F	T	F
F	T	F		T	F	T
F	F	F		F	F	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$				
T	T	T	T	T	T	T
T	F	T	F	F	T	F
F	T	F		T	F	T
F	F	F		F	F	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$				
T	T	T	T	T	T	T
T	F	T	F	F	T	F
F	T	F	T	T	F	T
F	F	F		F	F	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$				
T	T	T	T	T	T	T
T	F	T	F	F	T	F
F	T	F	T	T	F	T
F	F	F	T	F	F	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$					
T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	F
F	T	F	T	T	F	T	T
F	F	F	T	F	F	F	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$					
T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	F
F	T	F	T	T	F		T
F	F	F	T	F	F		F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Using the same technique we can fill in the full truth table for

$$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$					
T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	F
F	T	F	T	T	F	F	T
F	F	F	T	F	F	F	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$					
T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	F
F	T	F	T	T	F	F	T
F	F	F	T	F	F	F	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$						
T	T	F	T	T	T	T	T	T
T	F	T	F	F	T	F	F	F
F	T	F	T	T	F	F	T	T
F	F	F	T	F	F	F	F	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$						
T	T	F	T	T	T	T	T	T
T	F	T	T	F	F	T	F	F
F	T	F	T	T	F	F	F	T
F	F	F	T	F	F	F	F	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Using the same technique we can fill in the full truth table for

$$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$						
T	T	F	T	T	T	T	T	T
T	F	T	T	F	F	T	F	F
F	T	F	F	T	T	F	F	T
F	F		F	T	F	F	F	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$						
T	T	F	T	T	T	T	T	T
T	F	T	T	F	F	T	F	F
F	T	F	F	T	T	F	F	T
F	F	F	F	T	F	F	F	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Using the same technique we can fill in the full truth table for

$$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$							
T	T	F	T	T	T	T	T	T	T
T	F	T	T	F	F		T	F	F
F	T	F	F	T	T		F	F	T
F	F	F	F	T	F		F	F	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Using the same technique we can fill in the full truth table for

$$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$							
T	T	F	T	T	T	T	T	T	T
T	F	T	T	F	F	F	T	F	F
F	T	F	F	T	T	F	F	T	
F	F	F	F	T	F	F	F	F	

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$							
T	T	F	T	T	T	T	T	T	T
T	F	T	T	F	F	F	T	F	F
F	T	F	F	T	T	T	F	F	T
F	F	F	F	T	F	F	F	F	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Using the same technique we can fill in the full truth table for

$$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$							
T	T	F	T	T	T	T	T	T	T
T	F	T	T	F	F	F	T	F	F
F	T	F	F	T	T	T	F	F	T
F	F	F	F	T	F	T	F	F	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$							
T	T	F	T	T	T	T	T	T	T
T	F	T	T	F	F	F	T	F	F
F	T	F	F	T	T	T	F	F	T
F	F	F	F	T	F	T	F	F	F

The main column (in boldface) gives the truth-value of the whole sentence.

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	T	F	F	F	T

Validity

Let Γ be a set of sentences of \mathcal{L}_1 and ϕ a sentence of \mathcal{L}_1 .

Validity

Let Γ be a set of sentences of \mathcal{L}_1 and ϕ a sentence of \mathcal{L}_1 .

Definition

The argument with all sentences in Γ as premisses and ϕ as conclusion is *valid* if and only if there is no \mathcal{L}_1 -structure under which:

- (i) all sentences in Γ are true; and
- (ii) ϕ is false.

Validity

Let Γ be a set of sentences of \mathcal{L}_1 and ϕ a sentence of \mathcal{L}_1 .

Definition

The argument with all sentences in Γ as premisses and ϕ as conclusion is *valid* if and only if there is no \mathcal{L}_1 -structure under which:

- (i) all sentences in Γ are true; and
- (ii) ϕ is false.

Notation: when this argument is valid we write $\Gamma \models \phi$.

Validity

Let Γ be a set of sentences of \mathcal{L}_1 and ϕ a sentence of \mathcal{L}_1 .

Definition

The argument with all sentences in Γ as premisses and ϕ as conclusion is *valid* if and only if there is no \mathcal{L}_1 -structure under which:

- (i) all sentences in Γ are true; and
- (ii) ϕ is false.

Notation: when this argument is valid we write $\Gamma \models \phi$.

$\{P \rightarrow \neg Q, Q\} \models \neg P$ means that the argument whose premisses are $P \rightarrow \neg Q$ and Q , and whose conclusion is $\neg P$ is valid.

Also written: $P \rightarrow \neg Q, Q \models \neg P$

Worked example 3

We can use truth-tables to show that \mathcal{L}_1 -arguments are valid.

Worked example 3

We can use truth-tables to show that \mathcal{L}_1 -arguments are valid.

Example

Show that $\{P \rightarrow \neg Q, Q\} \models \neg P$.

Worked example 3

We can use truth-tables to show that \mathcal{L}_1 -arguments are valid.

Example

Show that $\{P \rightarrow \neg Q, Q\} \models \neg P$.

P	Q	$P \rightarrow \neg Q$	Q	$\neg P$
T	T	T F F T	T	F T
T	F	T T T F	F	F T
F	T	F T F T	T	T F
F	F	F T T F	F	T F

Worked example 3

We can use truth-tables to show that \mathcal{L}_1 -arguments are valid.

Example

Show that $\{P \rightarrow \neg Q, Q\} \models \neg P$.

P	Q	$P \rightarrow \neg Q$	Q	$\neg P$
T	T	T F F T	T	F T
T	F	T T T F	F	F T
F	T	F T F T	T	T F
F	F	F T T F	F	T F

Rows correspond to interpretations.

Worked example 3

We can use truth-tables to show that \mathcal{L}_1 -arguments are valid.

Example

Show that $\{P \rightarrow \neg Q, Q\} \models \neg P$.

P	Q	$P \rightarrow \neg Q$	Q	$\neg P$
T	T	T F T	T	F T
T	F	T T F	F	F T
F	T	F T T	T	T F
F	F	F T F	F	T F

Rows correspond to interpretations.

One needs to check that there is no row in which all the premisses are assigned T and the conclusion is assigned F.

Worked example 3

We can use truth-tables to show that \mathcal{L}_1 -arguments are valid.

Example

Show that $\{P \rightarrow \neg Q, Q\} \models \neg P$.

	P	Q	$P \rightarrow \neg Q$			Q	$\neg P$
▶	T	T	T	F	T	T	F T
	T	F	T	T	F	F	F T
	F	T	F	T	T	T	T F
	F	F	F	T	F	F	T F

Rows correspond to interpretations.

One needs to check that there is no row in which all the premisses are assigned T and the conclusion is assigned F.

Worked example 3

We can use truth-tables to show that \mathcal{L}_1 -arguments are valid.

Example

Show that $\{P \rightarrow \neg Q, Q\} \models \neg P$.

	P	Q	$P \rightarrow \neg Q$	Q	$\neg P$
	T	T	T F F T	T	F T
▶	T	F	T T T F	F	F T
	F	T	F T F T	T	T F
	F	F	F T T F	F	T F

Rows correspond to interpretations.

One needs to check that there is no row in which all the premisses are assigned T and the conclusion is assigned F.

Worked example 3

We can use truth-tables to show that \mathcal{L}_1 -arguments are valid.

Example

Show that $\{P \rightarrow \neg Q, Q\} \models \neg P$.

P	Q	$P \rightarrow \neg Q$	Q	$\neg P$
T	T	T F F T	T	F T
T	F	T T T F	F	F T
▶ F	T	F T F T	T	T F
F	F	F T T F	F	T F

Rows correspond to interpretations.

One needs to check that there is no row in which all the premisses are assigned T and the conclusion is assigned F.

Worked example 3

We can use truth-tables to show that \mathcal{L}_1 -arguments are valid.

Example

Show that $\{P \rightarrow \neg Q, Q\} \models \neg P$.

P	Q	$P \rightarrow \neg Q$	Q	$\neg P$
T	T	T F F T	T	F T
T	F	T T T F	F	F T
F	T	F T F T	T	T F
▶ F	F	F T T F	F	T F

Rows correspond to interpretations.

One needs to check that there is no row in which all the premisses are assigned T and the conclusion is assigned F.

Other logical notions

Definition

A sentence ϕ of \mathcal{L}_1 is *logically true* (a *tautology*) iff:

- ϕ is true under all \mathcal{L}_1 -structures.

Other logical notions

Definition

A sentence ϕ of \mathcal{L}_1 is *logically true* (a *tautology*) iff:

- ϕ is true under all \mathcal{L}_1 -structures.

e.g. $P \vee \neg P$, and $P \rightarrow P$ are tautologies.

Other logical notions

Definition

A sentence ϕ of \mathcal{L}_1 is *logically true* (a *tautology*) iff:

- ϕ is true under all \mathcal{L}_1 -structures.

e.g. $P \vee \neg P$, and $P \rightarrow P$ are tautologies.

Truth tables of tautologies

Every row in the main column is a T.

P	$P \vee \neg P$	$P \rightarrow P$
T	T T F T	T T T
F	F T T F	F T F

Definition

A sentence ϕ of \mathcal{L}_1 is a *contradiction* iff:

- ϕ is not true under any \mathcal{L}_1 -structure.

Definition

A sentence ϕ of \mathcal{L}_1 is a *contradiction* iff:

- ϕ is not true under any \mathcal{L}_1 -structure.

e.g. $P \wedge \neg P$, and $\neg(P \rightarrow P)$ are contradictions.

Definition

A sentence ϕ of \mathcal{L}_1 is a *contradiction* iff:

- ϕ is not true under any \mathcal{L}_1 -structure.

e.g. $P \wedge \neg P$, and $\neg(P \rightarrow P)$ are contradictions.

Truth tables of contradictions

Every row in the main column is an F.

P	$P \wedge \neg P$	$\neg (P \rightarrow P)$
T	T F F T	F T T T
F	F F T F	F F T F

Definition

Sentences ϕ and ψ are *logically equivalent* iff:

- ϕ and ψ are true in exactly the same \mathcal{L}_1 -structures.

Definition

Sentences ϕ and ψ are *logically equivalent* iff:

- ϕ and ψ are true in exactly the same \mathcal{L}_1 -structures.
- P and $\neg\neg P$ are logically equivalent.
- $P \wedge Q$ and $\neg(\neg P \vee \neg Q)$ are logically equivalent.

Definition

Sentences ϕ and ψ are *logically equivalent* iff:

- ϕ and ψ are true in exactly the same \mathcal{L}_1 -structures.
- P and $\neg\neg P$ are logically equivalent.
- $P \wedge Q$ and $\neg(\neg P \vee \neg Q)$ are logically equivalent.

Truth tables of logical equivalents

The truth-values in the main columns agree.

P	Q	$P \wedge Q$	$\neg(\neg P \vee \neg Q)$
T	T	T T	T F T F F T
T	F	T F F	F F T T T F
F	T	F F T	F T F T F T
F	F	F F F	F T F T T F

*Worked example 4***Example**

Show that the sentence $(P \rightarrow (\neg Q \wedge R)) \vee P$ is a tautology.

Worked example 4

Example

Show that the sentence $(P \rightarrow (\neg Q \wedge R)) \vee P$ is a tautology.

Method 1: Full truth table

- Write out the truth table for $(P \rightarrow (\neg Q \wedge R)) \vee P$.
- Check there's a T in every row of the main column.

Worked example 4

Example

Show that the sentence $(P \rightarrow (\neg Q \wedge R)) \vee P$ is a tautology.

Method 1: Full truth table

- Write out the truth table for $(P \rightarrow (\neg Q \wedge R)) \vee P$.
- Check there's a T in every row of the main column.

P	Q	R	$(P \rightarrow (\neg Q \wedge R)) \vee P$							
T	T	T	T	F	F	T	F	T	T	T
T	T	F	T	F	F	T	F	F	T	T
T	F	T	T	T	T	F	T	T	T	T
T	F	F	T	F	T	F	F	F	T	T
F	T	T	F	T	F	T	F	T	T	F
F	T	F	F	T	F	T	F	F	T	F
F	F	T	F	T	T	F	T	T	T	F
F	F	F	F	T	T	F	F	F	T	F

Worked example 4 (cont.)

Show that the sentence $(P \rightarrow (\neg Q \wedge R)) \vee P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P \rightarrow (\neg Q \wedge R)) \vee P$

Worked example 4 (cont.)

Show that the sentence $(P \rightarrow (\neg Q \wedge R)) \vee P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R		$(P \rightarrow (\neg Q \wedge R)) \vee P$
				F

Worked example 4 (cont.)

Show that the sentence $(P \rightarrow (\neg Q \wedge R)) \vee P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	\parallel	$(P \rightarrow (\neg Q \wedge R)) \vee P$
			\parallel	F

ϕ	\parallel	$\neg\phi$	\parallel	ϕ	ψ	\parallel	$(\phi \wedge \psi)$	\parallel	$(\phi \vee \psi)$	\parallel	$(\phi \rightarrow \psi)$	\parallel
T	\parallel	F	\parallel	T	T	\parallel	T	\parallel	T	\parallel	T	\parallel
T	\parallel	F	\parallel	T	F	\parallel	F	\parallel	T	\parallel	F	\parallel
F	\parallel	T	\parallel	F	T	\parallel	F	\parallel	T	\parallel	T	\parallel
F	\parallel	F	\parallel	F	F	\parallel	F	\parallel	F	\parallel	T	\parallel

Worked example 4 (cont.)

Show that the sentence $(P \rightarrow (\neg Q \wedge R)) \vee P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P \rightarrow (\neg Q \wedge R))$	$\vee P$
			F ₁	F

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \vee \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T	T
T	F	T	F	F	T	F
F	T	F	T	F	T	T
		F	F	F	F	T

Worked example 4 (cont.)

Show that the sentence $(P \rightarrow (\neg Q \wedge R)) \vee P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P \rightarrow (\neg Q \wedge R))$	$\vee P$
			F_1	$F \quad F_2$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \vee \psi)$	$(\phi \rightarrow \psi)$
T	F	T	T	T	T	T
T	F	T	F	F	T	F
F	T	F	T	F	T	T
F	T	F	F	F	F	T

Worked example 4 (cont.)

Show that the sentence $(P \rightarrow (\neg Q \wedge R)) \vee P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P \rightarrow (\neg Q \wedge R)) \vee P$
			F_1 F F_2

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \vee \psi)$	$(\phi \rightarrow \psi)$	
T	F	T	T	T	T	T	
T	F	F	F	F	T	F	
F	T	F	T	F	T	T	
F	T	F	F	F	F	T	

Worked example 4 (cont.)

Show that the sentence $(P \rightarrow (\neg Q \wedge R)) \vee P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P \rightarrow (\neg Q \wedge R)) \vee P$
			$\begin{array}{cc} T_3 & F_1 \\ & F \quad F_2 \end{array}$

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \vee \psi)$	$(\phi \rightarrow \psi)$	
T	F	T	T	T	T	T	
T	F	T	F	F	T	F	
F	T	F	T	F	T	T	
F	F	F	F	F	F	T	

Worked example 4 (cont.)

Show that the sentence $(P \rightarrow (\neg Q \wedge R)) \vee P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P \rightarrow (\neg Q \wedge R)) \vee P$
			T ₃ F ₁ F F ₂

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \vee \psi)$	$(\phi \rightarrow \psi)$	
T	F	T	T	T	T	T	
T	F	T	F	F	T	F	
F	T	F	T	F	T	T	
F	T	F	F	F	F	T	

Worked example 4 (cont.)

Show that the sentence $(P \rightarrow (\neg Q \wedge R)) \vee P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P \rightarrow (\neg Q \wedge R)) \vee P$
			? F ₁ F F ₂

ϕ	$\neg\phi$	ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \vee \psi)$	$(\phi \rightarrow \psi)$	
T	F	T	T	T	T	T	
T	F	T	F	F	T	F	
F	T	F	T	F	T	T	
F	T	F	F	F	F	T	