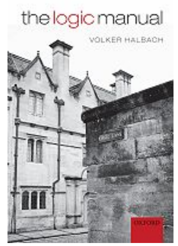
INTRODUCTION TO LOGIC

1 Sets, Relations, and Arguments

Volker Halbach

Pure logic is the ruin of the spirit.
Antoine de Saint-Exupéry



- *The Logic Manual*

web page for the book: http://logicmanual.philosophy.ox.ac.uk/

- *Exercises Booklet*
- *More Exercises* by Peter Fritz
- slides of the lectures
- worked examples
- past examination papers with solutions

Mark Sainsbury: *Logical Forms: An Introduction to Philosophical Logic*, Blackwell, second edition, 2001

Why logic?

Logic is the scientific study of valid argument.

- Philosophy is all about arguments and reasoning.
- Logic allows us to test validity rigorously.
- Modern philosophy assumes familiarity with logic.
- Used in linguistics, mathematics, computer science,...
- Helps us make fine-grained conceptual distinctions.
- Logic is compulsory.

Arguments

Definition
Sentences that are true or false are called declarative sentences.

In what follows I will focus exclusively on declarative sentences.

Definition
An argument consists of a set of declarative sentences (the premisses) and a declarative sentence (the conclusion) somehow marked as the concluded sentence.

Example

I'm not dreaming if I can see the computer in front of me. I can see the computer in front of me. Therefore I'm not dreaming.

'I'm not dreaming if I can see the computer in front of me' is a premiss.

'I can see the computer in front of me' is a premiss.

'I'm not dreaming' is the conclusion, which is marked by 'therefore'.

Occasionally the conclusion precedes the premisses or is found between premisses. The conclusion needn't be marked as such by 'therefore' or a similar phrase.

Alternative ways to express the argument:

Example

I'm not dreaming. For if I can see the computer in front of me I'm not dreaming, and I can see the computer in front of me.

Example

I'm not dreaming, if I can see the computer in front of me. Thus, I'm not dreaming. This is because I can see the computer in front of me.

The point of 'good' arguments is that the truth of the premisses guarantees the truth of the conclusion. Many arguments with this property exhibit certain patterns.

Example

I'm not dreaming if I can see the computer in front of me. I can see the computer in front of me. Therefore I'm not dreaming.

Example

Fiona can open the dvi-file if YAP is installed. YAP is installed. Therefore Fiona can open the dvi-file.

general form of both arguments

A if *B*. *B*. Therefore *A*.

Logicians are interested in the patterns of 'good' arguments that cannot take one from true premisses to a false conclusion.

Characterisation

An argument is logically (or formally) valid if and only if there is no interpretation under which the premisses are all true and the conclusion is false.

Example

Zeno is a tortoise. All tortoises are toothless. Therefore Zeno is toothless.

Example

Socrates is a man. All men are mortal. Therefore Socrates is mortal.

Features of logically valid arguments:

- The truth of the conclusion follows from the truth of the premisses independently what the subject-specific expressions mean. Whatever tortoises are, whoever Zeno is, whatever exists: if the premisses of the argument are true the conclusion will be true.
- The truth of the conclusion follows from the truth of the premisses purely in virtue of the 'form' of the argument and independently of any subject-specific assumptions.
- It's not possible that the premisses of a logically valid argument are true and its conclusion is false.
- In a logically valid argument the conclusion can be false (in that case at least one of its premisses is false).
- Validity does not depend on the meanings of subject-specific expressions.

The following argument isn't logically valid:

Example

Every EU citizen can enter the US without a visa. Max is a citizen of Sweden. Therefore Max can enter the US without a visa.

However, one can transform it into a logically valid argument by adding a premiss:

Example

Every EU citizen can enter the US without a visa. Max is a citizen of Sweden. Every citizen of Sweden is a EU citizen. Therefore Max can enter the US without a visa.

Characterisation (consistency)

A set of sentences is consistent if and only if there is a least one interpretation under which all sentences of the set are true.

Characterisation (logical truth)

A sentence is logically true if and only if it is true under any interpretation.

'All metaphysicians are metaphysicians.'

Characterisation (contradiction)

A sentence is a contradiction if and only if it is false under any interpretation.

'Some metaphysicians who are also ethicists aren't metaphysicians.'

I'll make these notions precise for the formal languages or propositional and predicate logic.

Sets

The following is not really logic in the strict sense but we'll need it later and it is useful in other areas as well.

Characterisation

A set is a collection of objects.

The objects in the set are the elements of the set.

There is a set that has exactly all books as elements.
There is a set that has Volker Halbach as its only element.

Sets are identical if and only if they have the same elements.

Example

The set of all animals with kidneys and the set of all animals with a heart are identical, because exactly those animals that have kidneys also have a heart and vice versa.

The claim 'a is an element of S' can be written as '$a \in S$'. One also says 'S contains a' or 'a is in S'.

There is exactly one set with no elements. The symbol for this set is '$\varnothing$'.

The set {Oxford, $\varnothing$, Volker Halbach} has as its elements exactly three things: Oxford, the empty set $\varnothing$, and me.

Here is another way to denote sets:

$$\{ d : d \text{ is an animal with a heart} \}$$

is the set of all animals with a heart. It has as its elements exactly all animals with a heart.

Example

{Oxford, $\varnothing$, Volker Halbach} = {Volker Halbach, Oxford, $\varnothing$ }

Example

{the capital of England, Munich} = {London, Munich, the capital of England}

Example

Mars $\in \{ d : d \text{ is a planet} \}$

Example

$\varnothing \in \{\varnothing\}$

Relations

The set {London, Munich} is the same set as {Munich, London}.

The ordered pair ⟨London, Munich⟩ is different from the ordered pair ⟨Munich, London⟩.

Ordered pairs are identical if and only if the agree in their first and second components, or more formally:

$\langle d, e \rangle = \langle f, g \rangle$ iff ($d = f$ and $e = g$)

The abbreviation 'iff' stands for 'if and only if.'

There are also triples (3-tuples) like ⟨London, Munich, Rome⟩, quadruples, 5-tuples, 6-tuples etc.

Definition

A set is a binary relation if and only if it contains only ordered pairs.

The empty set ∅ doesn't contain anything that's not an ordered pair; therefore it's a relation.

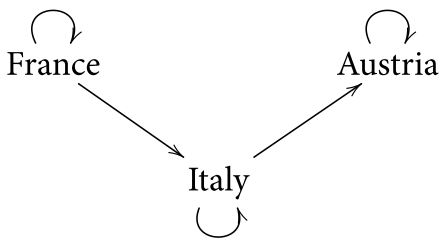
Example

The relation of *being a bigger city than* is the set {⟨London, Munich⟩, ⟨London, Birmingham⟩, ⟨Paris, Milan⟩…}.

The following set is a binary relation:

{⟨France, Italy⟩, ⟨Italy, Austria⟩, ⟨France, France⟩, ⟨Italy, Italy⟩, ⟨Austria, Austria⟩}

Some relations can be visualised by diagrams. Every pair corresponds to an arrow:

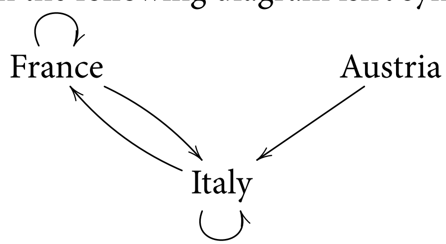


I'll mention only some properties of relations.

Definition

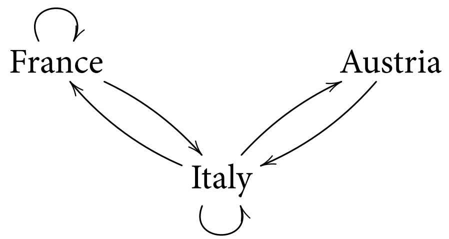
A binary relation R is symmetric iff for all d, e: if $\langle d, e \rangle \in R$ then $\langle e, d \rangle \in R$.

The relation with the following diagram isn't symmetric:



The pair ⟨Austria, Italy⟩ is in the relation, but the pair ⟨Italy, Austria⟩ isn't.

The relation with the following diagram is symmetric.



Definition

A binary relation is transitive iff for all d, e, f: if $\langle d, e \rangle \in R$ and $\langle e, f \rangle \in R$, then also $\langle d, f \rangle \in R$

In the diagram of a transitive relation there is for any two-arrow way from an point to a point a direct arrow.

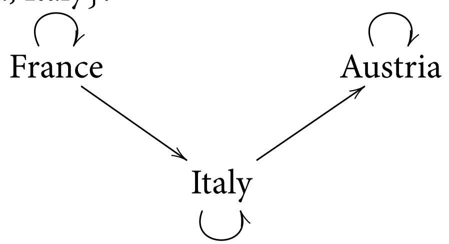
This is the diagram of a relation that's not transitive:

This is the diagram of a relation that is transitive:

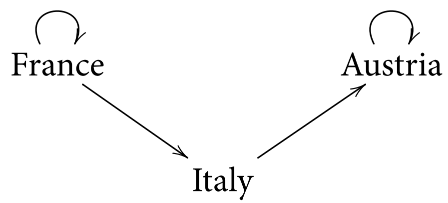
Definition

A binary relation R is reflexive on a set S iff for all d in S the pair $\langle d, d \rangle$ is an element of R.

The relation with the following diagram is reflexive on the set $\{$France, Austria, Italy$\}$.



The relation with the following diagram is not reflexive on $\{$France, Austria, Italy$\}$, but reflexive on $\{$France, Austria$\}$:

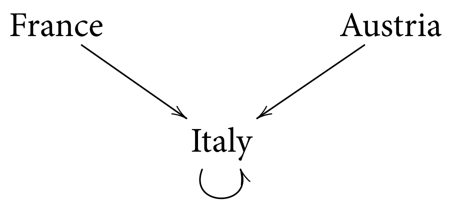


Functions

Definition

A binary relation R is a function iff for all d, e, f: if $\langle d, e \rangle \in R$ and $\langle d, f \rangle \in R$ then $e = f$.

The relation with the following diagram is a function:



There is at most one arrow leaving from every point in the diagram of a function.

Example

The set of all ordered pairs $\langle d, e \rangle$ such that e is mother of d is a function.

This justifies talking about *the* mother of so-and-so.

You might know examples of the following kind from school:

Example

The set of all pairs $\langle d, d^2 \rangle$ where d is some real number is a function.

One can't write down all the pairs, but the function would look like this: $\{\langle 2, 4 \rangle, \langle 1, 1 \rangle, \langle 5, 25 \rangle, \langle \frac{1}{2}, \frac{1}{4} \rangle \ldots\}$

One also think of a function as something that yields an 'output', e.g. 25 when given an input, e.g. 5, or that 'assigns the value 25 to the argument 5'.