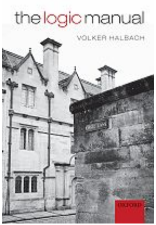
INTRODUCTION TO LOGIC

1 Sets, Relations, and Arguments

Volker Halbach

Pure logic is the ruin of the spirit.
Antoine de Saint-Exupéry



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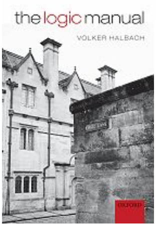

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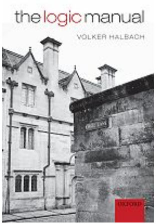

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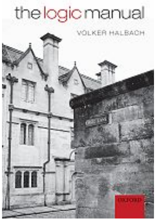

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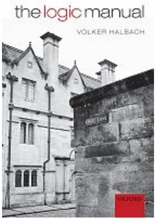

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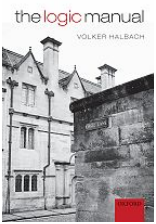

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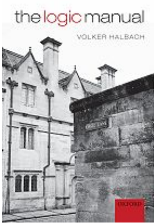

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Mark Sainsbury: *Logical Forms: An Introduction to Philosophical Logic*, Blackwell, second edition, 2001

Why logic?

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- Logic is compulsory.

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An argument consists of a set of declarative sentences (the premisses) and a declarative sentence (the conclusion) somehow marked as the concluded sentence.

Example

I'm not dreaming if I can see the computer in front of me. I can see the computer in front of me. Therefore I'm not dreaming.

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'I'm not dreaming' is the conclusion, which is marked by 'therefore'.

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Example

I'm not dreaming, if I can see the computer in front of me. Thus, I'm not dreaming. This is because I can see the computer in front of me.

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Fiona can open the dvi-file if YAP is installed. YAP is installed. Therefore Fiona can open the dvi-file.

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general form of both arguments

A if *B*. *B*. Therefore *A*.

Logicians are interested in the patterns of 'good' arguments that cannot take one from true premisses to a false conclusion.

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Example

Socrates is a man. All men are mortal. Therefore Socrates is mortal.

Features of logically valid arguments:

- The truth of the conclusion follows from the truth of the premisses independently what the subject-specific expressions mean. Whatever tortoises are, whoever Zeno is, whatever exists: if the premisses of the argument are true the conclusion will be true.

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The following argument isn't logically valid:

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However, one can transform it into a logically valid argument by adding a premiss:

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Every EU citizen can enter the US without a visa. Max is a citizen of Sweden. Every citizen of Sweden is a EU citizen. Therefore Max can enter the US without a visa.

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Characterisation (logical truth)

A sentence is logically true if and only if it is true under any interpretation.

'All metaphysicians are metaphysicians.'

Characterisation (contradiction)

A sentence is a contradiction if and only if it is false under any interpretation.

‘Some metaphysicians who are also ethicists aren’t metaphysicians.’

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I'll make these notions precise for the formal languages or propositional and predicate logic.

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There is a set that has exactly all books as elements.
There is a set that has Volker Halbach as its only element.

Sets are identical if and only if they have the same elements.

Example

The set of all animals with kidneys and the set of all animals with a heart are identical, because exactly those animals that have kidneys also have a heart and vice versa.

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Here is another way to denote sets:

$$\{\, d : \ d \text{ is an animal with a heart} \,\}$$

is the set of all animals with a heart. It has as its elements exactly all animals with a heart.

Example

$\{$Oxford, $\varnothing$, Volker Halbach$\}$ = $\{$Volker Halbach, Oxford, $\varnothing$ $\}$

Example

{Oxford, ∅, Volker Halbach} = {Volker Halbach, Oxford, ∅ }

Example

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$\varnothing \in \{\varnothing\}$

Relations

The set {London, Munich} is the same set as {Munich, London}.

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Ordered pairs are identical if and only if the agree in their first and second components, or more formally:

$$\langle d, e \rangle = \langle f, g \rangle \text{ iff } (d = f \text{ and } e = g)$$

The abbreviation 'iff' stands for 'if and only if'.

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There are also triples (3-tuples) like $\langle$London, Munich, Rome$\rangle$, quadruples, 5-tuples, 6-tuples etc.

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Example

The relation of *being a bigger city than* is the set $\{\langle$London, Munich$\rangle$, $\langle$London, Birmingham$\rangle$, $\langle$Paris, Milan$\rangle \ldots\}$.

The following set is a binary relation:

$$\{\langle \text{France}, \text{Italy} \rangle, \langle \text{Italy}, \text{Austria} \rangle, \langle \text{France}, \text{France} \rangle, \langle \text{Italy}, \text{Italy} \rangle, \langle \text{Austria}, \text{Austria} \rangle\}$$

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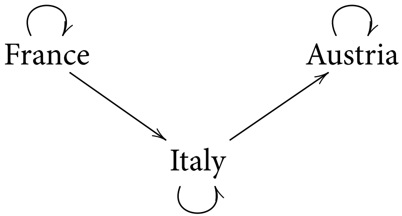
Some relations can be visualised by diagrams. Every pair corresponds to an arrow:



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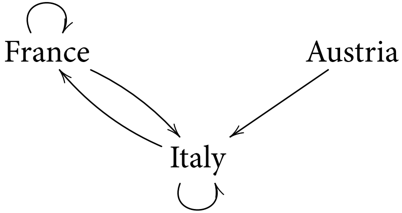
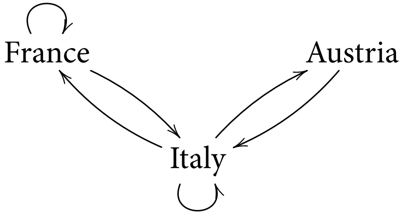
The relation with the following diagram isn't symmetric:



The pair ⟨Austria, Italy⟩ is in the relation, but the pair ⟨Italy, Austria⟩ isn't.

The relation with the following diagram is symmetric.



Definition

A binary relation is transitive iff for all d, e, f: if $\langle d, e \rangle \in R$ and $\langle e, f \rangle \in R$, then also $\langle d, f \rangle \in R$

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This is the diagram of a relation that's not transitive:

$$\text{France} \longrightarrow \text{Austria} \longrightarrow \text{Italy}$$

This is the diagram of a relation that is transitive:

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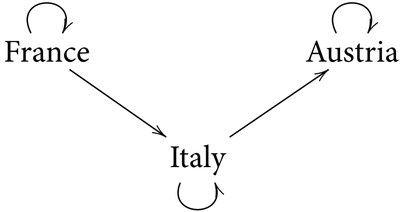
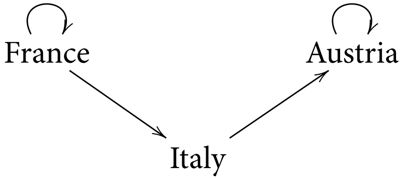
The relation with the following diagram is reflexive on the set $\{$France, Austria, Italy$\}$.



The relation with the following diagram is not reflexive on {France, Austria, Italy}, but reflexive on {France, Austria}:



Functions

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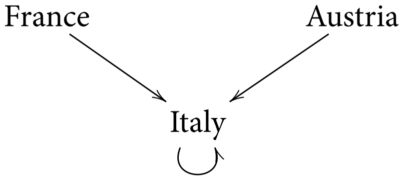
The relation with the following diagram is a function:



There is at most one arrow leaving from every point in the diagram of a function.

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This justifies talking about *the* mother of so-and-so.

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You might know examples of the following kind from school:

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The set of all pairs $\langle d, d^2\rangle$ where d is some real number is a function.

One can't write down all the pairs, but the function would look like this: $\{\langle 2, 4\rangle, \langle 1, 1\rangle, \langle 5, 25\rangle, \langle \frac{1}{2}, \frac{1}{4}\rangle \ldots\}$

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One also think of a function as something that yields an 'output', e.g. 25 when given an input, e.g. 5, or that 'assigns the value 25 to the argument 5'.