SAMPLE PAPER

# INTRODUCTION TO PHILOSOPHY

# SECTION A: LOGIC

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*I thank the author of the 2008 Trinity Term paper, Andrew Bacon, and Kentaro Fujimoto for their help in preparing this sample paper. The text for formalisation in propositional logic in Question 1 is taken from the 2008 Trinity Term paper.*

*In this version I have included hints for answering the questions. My hints are not intended as complete answers; rather they could be the core of a full answer. In particular, in most cases my incomplete answer would have to be supplemented by additional comments and explanations. In many cases completely different approaches can lead to an equally good if not better answer.*

1. (a) If a set of English sentences is propositionally consistent, is it then also consistent (simpliciter)? Substantiate your answer.

   (b) If the conclusion of an English argument is a tautology can the argument fail to be logically valid?

   (c) Show that the sentences in the following paragraph make up a propositionally inconsistent set by symbolizing them and by showing that the set of formalisations is semantically inconsistent (either by the truth table method or by a proof in Natural Deduction). Specify your dictionary carefully, and comment on anything difficult or otherwise noteworthy about the symbolization.

   > The planet Venus is visible in the evening—hence the use of a name 'Hesperus' (the Evening Star)—and it's visible a second time in the morning—hence the use of a name 'Phosphorus' (the Morning Star). This implies that 'Hesperus' and 'Phosphorus' are different names for the very same planet, which in turn implies that the sentence 'Hesperus is the same planet as Phosphorus' is true. However, it's possible—in fact, it's no doubt actually the case—that there are

people who are good enough logicians to know that ‘Hesperus is the same planet as Hesperus’ is true and yet don’t know that ‘Hesperus is the same planet as Phosphorus’ is true. Now, either ‘Hesperus is the same planet as Hesperus’ and ‘Hesperus is the same planet as Phosphorus’ mean the same thing or they don’t. If they do, then it must after all be the case that anyone who’s a good enough logician to know that ‘Hesperus is the same planet as Hesperus’ is true will also know that ‘Hesperus is the same planet as Phosphorus’ is true. On the other hand, if these sentences mean different things, then ‘Hesperus’ and ‘Phosphorus’ will mean different things. But they’ll only mean different things if they’re names for different planets.

2. (a) Establish the following claims by means of proofs in the system of Natural Deduction:

(i) $P \wedge Q \wedge R \vdash (P \to P_1) \to ((P_1 \to Q_1) \to Q_1)$

*Answer.*

Proof:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(P \wedge Q) \wedge R}{P \wedge Q}}{P} \qquad [P \to P_1]}{P_1} \qquad [P_1 \to Q_1]}{Q_1}}{(P_1 \to Q_1) \to Q_1}}{(P \to P_1) \to ((P_1 \to Q_1) \to Q_1)}$$

(ii) $Pa \wedge Qc, \forall x\,(Px \to Rxa) \vdash \exists y\,Ryy \vee \neg\exists z\,Qz$

*Answer.*

Proof:

$$\dfrac{\dfrac{\dfrac{\dfrac{Pa \wedge Qc}{Pa} \qquad \dfrac{\forall x\,(Px \to Rxa)}{Pa \to Raa}}{Raa}}{\exists y\,Ryy}}{\exists y\,Ryy \vee \neg\exists z\,Qz}$$

(iii) $\forall x\,\exists y\,\neg Rxy \vdash \forall x\,\exists y\,\exists z\,(\neg Rxy \wedge \neg Ryz)$

*Answer.*

Proof:

$$
\dfrac{\dfrac{\dfrac{\forall x\,\exists y\,\neg Rxy}{\exists y\,\neg Ray} \qquad \dfrac{\dfrac{\forall x\,\exists y\,\neg Rxy}{\exists y\,\neg Rby} \qquad \dfrac{\dfrac{\dfrac{[\neg Rab] \qquad [\neg Rbc]}{\neg Rab \land \neg Rbc}}{\exists z\,(\neg Rab \land \neg Rbz)}}{\exists y\,\exists z\,(\neg Ray \land \neg Ryz)}}{\exists y\,\exists z\,(\neg Ray \land \neg Ryz)}}{\exists y\,\exists z\,(\neg Ray \land \neg Ryz)}}{\forall x\,\exists y\,\exists z\,(\neg Rxy \land \neg Ryz)}
$$

(b) Assume that $\phi$ and all elements of $\Gamma$ are $\mathcal{L}_2$-sentences. Using the completeness or soundness theorem for $\mathcal{L}_2$ show that $\Gamma \nvDash \phi$ implies $\Gamma \nvdash \phi$.

*Answer.* By the soundness theorem $\Gamma \vdash \phi$ implies $\Gamma \vDash \phi$. Thus, if $\Gamma \nvDash \phi$ it follows that $\Gamma \nvDash \phi$.

(c) Establish the following claims by means of counterexamples.

(i) $P \land Q \land R \nvDash (P \to P_1) \to (P_1 \to Q_1)$

*Answer.*

| $P$ | $Q$ | $R$ | $P_1$ | $Q_1$ | $(P \land Q) \land R$ | $(P \to P_1) \to (P_1 \to Q_1)$ |
|---|---|---|---|---|---|---|
| T | T | T | T | F | T T T T T | T T T F T F F |

So any $\mathcal{L}_1$-structure $\mathcal{A}$ satisfying the following condition is a counterexample:

$$
\begin{aligned}
|P|_{\mathcal{A}} &= \mathrm{T}\\
|Q|_{\mathcal{A}} &= \mathrm{T}\\
|R|_{\mathcal{A}} &= \mathrm{T}\\
|P_1|_{\mathcal{A}} &= \mathrm{T}\\
|Q_1|_{\mathcal{A}} &= \mathrm{F}
\end{aligned}
$$

(ii) $Pa \land Qc, \forall x\,(Px \to Rxa) \nvDash \exists y\,Ryy \land \neg\exists z\,Qz$ *Answer.* Any structure with $\{1\}$ as domain and satisfying the following conditions is a counterexample (I am saying 'any' because I don't specify the semantic values for the other constants and predicate letters, which don't matter here):

$$
\begin{aligned}
|a|_{\mathcal{B}} &= 1\\
|c|_{\mathcal{B}} &= 1\\
|P|_{\mathcal{B}} &= \{1\}\\
|Q|_{\mathcal{B}} &= \{1\}\\
|R|_{\mathcal{B}} &= \{\langle 1,1\rangle\}
\end{aligned}
$$

You should add an explanation why this counterexample works.

(iii) $\forall x\, \exists y\, \neg Rxy \nvDash \forall x\, \exists y\, \exists z\, (\neg Rxy \wedge \neg Ryz \wedge \neg Rzx)$ *Answer.* Let $\mathcal{C}$ be a structure with domain $\{1, 2\}$ assigning a semantic value to $R^2$ in the following way:

$$|R|_{\mathcal{C}} = \{\langle 1, 1\rangle, \langle 2, 2\rangle\}$$

The values of the constants and the other predicate letters don't matter.

You should add an explanation why this counterexample works.

3. (a) Formalise the following sentences in predicate logic using the following dictionary noting any difficulties of points of interest:

$P$: … is a pen
$Q$: … owns …
$R$: … is red
$P_1$: … is green
$a$: John

(i) Some pens are red, some are not.

*Answer.* $\exists x\, (Px \wedge Rx) \wedge \exists x\, (Px \wedge \neg Rx)$

(ii) John owns a pen that isn't red, but he also owns a red pen. *Answer.* $\exists x\, (Px \wedge \neg Rx \wedge Qax) \wedge \exists x\, (Px \wedge Rx \wedge Qax)$

(iii) John owns only red or green pens.

*Answer.* This sentence is probably ambiguous. It could mean that John owns only red pens (but no green) or only green pens (but no red pen). The formalisation would be:

$$\forall x\, (Px \wedge Qax \to Rx) \vee \forall x\, (Px \wedge Qax \to P_1x)$$

It could also mean that all of John's pens are red or green.

$$\forall x\, (Px \wedge Qax \to Rx \vee P_1x)$$

(iv) Unless one of Johns pens is red, all of his pens are green.

*Answer.* $\exists x\, (Px \wedge Rax \wedge Rx) \vee \forall x\, (Px \wedge Rax \to P_1x)$

(v) John owns something green or red which isn't a pen.

*Answer.* $\exists x\, (Qax \wedge (Rx \vee P_1x) \wedge \neg Px)$

(b) Show that the following argument is valid in predicate logic noting any difficulties or points of interest.

> Some book authors haven't written a novel. Therefore there are books that aren't novels.

*Answer.*
$P$: … is book
$Q$: … is a novel
$R$: … has written …

For the formalisation I assume that having written something and being the author of it amounts to the same. So a book author is somebody who has written a book.

The premiss 'Some book authors haven't written a novel' is translated in the following way:

$$\exists x\,(\exists y\,(Rxy \land Py) \land \neg\exists y\,(Rxy \land Qy))$$

The formalisation of the conclusion 'There are books that aren't novels' is as follows:

$$\exists x\,(Px \land \neg Qx)$$

The claim

$$\forall x\,(Qx \to Px), \exists x\,(\exists y\,(Rxy \land Py) \land \neg\exists y\,(Rxy \land Qy)) \vdash \exists x\,(Px \land \neg Qx)$$

can be established with the proof on the next page.

4. (a) Explain why the following attempted proofs are not correct proofs in the system of Natural Deduction. Note all steps that are not correct. Give complete correct proofs for each of the claims.

   (i) $P \to Q \vdash \neg P \lor Q$

$$\dfrac{\dfrac{\dfrac{\dfrac{[P] \qquad P \to Q}{Q}}{\neg P \lor Q} \qquad [\neg(\neg P \lor Q)]}{\neg P}}{\neg P \lor Q}$$

   *Answer.* The discharging of the occurrence of $\neg(\neg P \lor Q)$ isn't covered by any rule in the proof. However, it can be discharged by assuming the sentence again and then applying ¬Elim. Proof:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[P] \qquad P \to Q}{Q}}{\neg P \lor Q} \qquad [\neg(\neg P \lor Q)]}{\neg P}}{\neg P \lor Q} \qquad [\neg(\neg P \lor Q)]}{\neg P \lor Q}\ \neg\text{Elim}$$

$$
\dfrac{\exists x\,(\exists y\,(Rxy \land Py) \land \neg\exists y\,(Rxy \land Qy)) \qquad \dfrac{\dfrac{[\exists y\,(Ray \land Py) \land \neg\exists y\,(Ray \land Qy)]}{\exists y\,(Ray \land Py)} \qquad \dfrac{\dfrac{\dfrac{[Rab \land Pb]}{Pb} \qquad \dfrac{\dfrac{\dfrac{\dfrac{[Rab \land Pb]}{Rab} \qquad [Qb]}{Rab \land Qb}}{\exists y\,(Ray \land Qy)} \qquad \dfrac{[\exists y\,(Ray \land Py) \land \neg\exists y\,(Ray \land Qy)]}{\neg\exists y\,(Ray \land Qy)]}}{\neg Qb}}{Pb \land \neg Qb}}{\exists x\,(Px \land \neg Qx)}}{\exists x\,(Px \land \neg Qx)}}{\exists x\,(Px \land \neg Qx)}
$$

(ii) $\exists x\, Q_2xx \vdash \exists z\, Q_2zz$

$$\dfrac{\dfrac{\exists x\, Q_2xx}{Q_2aa}}{\exists z\, Q_2zz}$$

*Answer.* The first step isn't covered by any rule. I can be replaced by an application of the rule for eliminating $\exists$ in the following way:

$$\dfrac{\exists x\, Q_2xx \qquad \dfrac{Q_2aa}{\exists z\, Q_2zz}}{\exists z\, Q_2zz}\;\exists\text{Elim}$$

(iii) $\forall x\, \exists y\, (Qxy \wedge Pyx) \vdash \exists x\, \exists y\, \exists z\, (Qxy \wedge Pzy)$

$$\dfrac{\dfrac{\dfrac{\forall x\, \exists y\, (Qxy \wedge Pyx)}{\exists y\, (Qay \wedge Pya)} \qquad \dfrac{\dfrac{\forall x\, \exists y\, (Qxy \wedge Pyx)}{\exists y\, (Qby \wedge Pyb)} \qquad \dfrac{\dfrac{\dfrac{\dfrac{[Qab \wedge Pba]}{Qab} \qquad \dfrac{[Qbc \wedge Pcb]}{Pcb}}{Qab \wedge Pcb}}{\exists x\, (Qxb \wedge Pcb)}}{\exists x\, \exists y\, (Qxy \wedge Pcy)}}{\exists x\, \exists y\, (Qxy \wedge Pcy)}\;\exists\text{Elim}}{\exists x\, \exists y\, (Qxy \wedge Pcy)}}{\exists x\, \exists y\, \exists z\, (Qxy \wedge Pzy)}$$

*Answer.* The marked application of $\exists$Elim does not conform to the formulation of the rule because the sentence $\exists x\, \exists y\, (Qxy \wedge Pcy)$ contains the constant $c$. Moreover, the last step isn't a correct application of $\exists$Intro because the existential quantifier can be introduced only as the beginning of a sentence. The following proof is correct:

$$\dfrac{\dfrac{\forall x\, \exists y\, (Qxy \wedge Pyx)}{\exists y\, (Qay \wedge Pya)} \qquad \dfrac{\dfrac{\forall x\, \exists y\, (Qxy \wedge Pyx)}{\exists y\, (Qby \wedge Pyb)} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[Qab \wedge Pba]}{Qab} \qquad \dfrac{[Qbc \wedge Pcb]}{Pcb}}{Qab \wedge Pcb}}{\exists z\, (Qab \wedge Pzb)}}{\exists y\, \exists z\, (Qay \wedge Pzy)}}{\exists x\, \exists y\, \exists z\, (Qxy \wedge Pzy)}}{\exists x\, \exists y\, \exists z\, (Qxy \wedge Pzy)}}{\exists x\, \exists y\, \exists z\, (Qxy \wedge Pzy)}$$

(b) State the rule $\neg$Intro for the introduction of $\neg$. If there is a proof of $\phi$ from undischarged assumptions in $\Gamma$ in the system of Natural Deduction then there is also a proof of $\phi$ from undischarged assumptions in $\Gamma$ without an application of $\neg$Intro. Show why $\neg$Intro is dispensable.

*Answer.* The rule $\neg$Intro is formulated as follows:

> The result of appending a sentence $\neg\phi$ to a proof of $\psi$ and a proof of $\neg\psi$ and of discharging all assumptions of $\phi$ in both proofs is a proof of $\neg\phi$.

Assume there is only one application of ¬Intro in the proof, replace the part of the proof in the following way:

$$\dfrac{\begin{array}{c}[\phi]\\ \vdots\\ \psi\end{array}\qquad\begin{array}{c}[\phi]\\ \vdots\\ \neg\psi\end{array}}{\neg\phi}$$

by a subproof of the following shape where $\neg\neg\phi$ is a new assumption, which gets discharged in the last line in accordance with ¬Elim:

$$\dfrac{\begin{array}{c}\neg\text{Elim}\,\dfrac{[\neg\neg\phi]\qquad[\neg\phi]}{\phi}\\ \vdots\\ \psi\end{array}\qquad\begin{array}{c}\dfrac{[\neg\neg\phi]\qquad[\neg\phi]}{\phi}\,\neg\text{Elim}\\ \vdots\\ \neg\psi\end{array}}{\neg\phi}\,\neg\text{Elim}$$

If the proof contains more applications of ¬Intro, start with the smallest subproof with ¬Intro and eliminate ¬Intro using the trick above. Then replace larger and larger subproofs until all applications of ¬Intro have diappeared.

5. (a) Determine for each of the following relations

- whether it is reflexive on the set of all $\mathcal{L}_1$-sentences,
- whether it is symmetric,
- whether it is antisymmetric,
- whether it is asymmetric, and
- whether it is transitive.

Substantiate your answers. In the following $\phi$ and $\psi$ are understood to be $\mathcal{L}_1$-sentences.

(i) The set of all pairs $\langle\phi,\psi\rangle$ such that $\phi \vDash \psi$

*Answer.* reflexive on the set of all $\mathcal{L}_1$-sentences, transitive, but neither symmetric nor asymmetric nor antisymmetric.

(ii) The set of all pairs $\langle\phi,\psi\rangle$ such that $\phi \vDash \neg\psi$

*Answer.* not reflexive on $\mathcal{L}_1$, symmetric, not asymmetric, not antisymmetric, not transitive

(iii) The set of all pairs $\langle \phi, \psi \rangle$ such that $\phi \wedge \neg\phi \vDash \psi$

*Answer.* reflexive on $\mathcal{L}_1$, symmetric, but neither asymmetric nor antisymmetric, transitive

(b) What is a function? Determine for each of the following relations whether it is a function or not. Substantiate your answers.

(i) The set of all pairs $\langle d, e \rangle$ such that $d$ is a person and $e$ is $d$'s head

*Answer.* This is a function.

(ii) The set of all pairs $\langle e, d \rangle$ such that $d$ is a person and $e$ is $d$'s head

*Answer.* This is a function.

(iii) The set of all pairs $\langle d, e \rangle$ such that $d$ is a person and $e$ is one of $e$'s toes

*Answer.* This isn't a function.

(iv) The set of all pairs $\langle e, d \rangle$ such that $d$ is a person and $e$ is one of $e$'s toes

*Answer.* This is a function (I ignore siamese twins and similar problematic cases).

(c) Answer each of the following questions and give a reason for your answer either by providing an example having the properties in question or by proving that such a relation cannot exist.

(i) Is there a relation that is symmetric and asymmetric?

*Answer.* Yes, the empty relation is symmetric and asymmetric.

(ii) Is there a function that is reflexive on the set of all Oxford colleges and that is not transitive if all components of ordered pairs in the relation are Oxford colleges?

*Answer.* No. Assume that there is such a relation $R$. As it's not transitive there are colleges $d$, $e$, and $f$ such that $\langle d, e \rangle \in R$ and $\langle e, f \rangle \in R$ but $\langle d, f \rangle \notin R$. Since $R$ is reflexive on the set of all planets, $\langle d, d \rangle \in R$. As $R$ is a function and $\langle d, e \rangle \in R$, one has $d = e$; similarly, $e = f$. So $d$, $e$, and $f$ are all the same college. This contradicts the assumption $\langle d, f \rangle \notin R$. So there cannot be such a function that is reflexive on the set of all planets but not transitive.

(iii) Is there a relation containing more than two ordered pairs that is asymmetric, transitive, and a function?

*Answer.* Yes. The relation

$$\{\langle 1, 2 \rangle, \langle 3, 4 \rangle, \langle 5, 6 \rangle\}$$

is such a function.

6. (a) How is an $\mathcal{L}_2$-structure defined?

Answer. An $\mathcal{L}_2$-structure is an ordered pair $\langle D, I\rangle$ where $D$ is some non-empty set and $I$ is a function from the set of all constants, sentence letters and predicate letters such that the value of every constant is an element of $D$, the value of every sentence letter is a truth-value T or F, and the value of every $n$-ary predicate letter is an $n$-ary relation.

(b) Provide counterexamples that establish the following claims. You don't have to prove that the premisses are true in the $\mathcal{L}_2$-structure and that the conclusion is false.

(i) $\forall x\, Rxx \nvDash \exists x\, \forall y\, Rxy$

*Answer.* Let $\mathcal{A}$ be an $\mathcal{L}_2$-structure with domain $\{1, 2\}$ satisfying the following conditions:

$$|R|_{\mathcal{A}} = \{\langle 1,1\rangle, \langle 2,2\rangle\}$$

The structure $\mathcal{A}$ can assign any values to constants and predicate letters other than $R^2$.

(ii) $\exists x\, (Px \land \exists y\, Rxy), \forall x\, (Px \to \neg\exists z\, (Rxz \land Pz)) \nvDash Q$

*Answer.* Let $\mathcal{B}$ be an $\mathcal{L}_2$-structure with domain $\{1, 2\}$ satisfying the following conditions:

$$|R|_{\mathcal{B}} = \{\langle 1,2\rangle\}$$
$$|P|_{\mathcal{B}} = \{1\}$$
$$|Q|_{\mathcal{B}} = \mathrm{F}$$

The structure $\mathcal{B}$ can assign any values to constants and predicate letters other than $R^2$, $P^1$, and the sentence letter $Q$.

(c) Consider an $\mathcal{L}_2$-structure $\mathcal{S}$ with the domain $D_{\mathcal{S}}$ and the following semantic values of $a$ and $R$:

$$D_{\mathcal{S}} = \{\text{Europe, Asia, Australia}\}$$
$$|a|_{\mathcal{S}} = \text{Europe}$$
$$|R|_{\mathcal{S}} = \{\langle \text{Australia, Europe}\rangle, \langle \text{Europe, Asia}\rangle, \langle \text{Australia, Asia}\rangle\}$$

Are the following sentences true or false in this structure? Justify your answers as fully as possible.

(i) $\exists x\, Rax$

*Answer.* This sentence is true in $\mathcal{S}$. Let $\alpha$ be a variable assignment that assigns Asia to $x$ and reason as follows:

$\langle \text{Europe}, \text{Asia} \rangle \in \{\langle \text{Australia}, \text{Europe} \rangle, \langle \text{Europe}, \text{Asia} \rangle, \langle \text{Australia}, \text{Asia} \rangle\}$
$\langle |a|_{\mathcal{S}}, |x|^{\alpha}_{\mathcal{S}} \rangle \in |R|_{\mathcal{S}}$
$|Rax|^{\alpha}_{\mathcal{S}} = \mathrm{T}$
$|\exists x, Rax|_{\mathcal{S}} = \mathrm{T}$

In the penultimate step I have dropped the variable assignment as the semantic value of sentences does is not affected by variable assignments.

(ii) $\forall x\, Rax \to (\neg Raa \lor \forall x\, \forall y\, Rxy)$

*Answer.* This sentence is true in $\mathcal{S}$. I don't give the full proof, but $\neg Raa$ is true. Then one can use propositional logic only to proof the truth of the sentence.

(iii) $\forall x\, \exists y\, Rxy$

*Answer.* This sentence is false in $\mathcal{S}$. I don't give a proof, but there is not pair $\langle \text{Asia}, d \rangle$ in $|R|_{\mathcal{S}}$.

(iv) $\exists x\, Rxx$

*Answer.* This sentence is false in $\mathcal{S}$. I don't give a full proof, but the reason is that no pair $\langle d, d \rangle$ is in $|R|_{\mathcal{S}}$.

7. (a) For each of the sentences below explain the way in which it is ambiguous. If possible, reveal the ambiguity by formalising the sentence in two (or more) different ways using the same dictionary in each of its formalisations.

(i) Some oak species can be found on every continent.

*Answer.* This is a case of scope ambiguity: it means either that there is an oak species that is found on every continent *or* it means that on every continent some oak species is found (but not necessarily the same on all continents).

A. $\exists x\, (Px \land \forall y\, (Qy \to Rxy)))$

B. $\forall y\, (Qy \to \exists x(Px \land Rxy)))$

$P$: … is an oak species
$R$: … is found on …
$Q$: … is a continent

(ii) Tom can't find the table.

*Answer.* At least a lexical ambiguity: table as a piece of furniture or table in a book. 'the table' is a definite description and formalised accordingly.

A. $\exists x\,(Px \wedge \forall y\,(Py \rightarrow y = x) \wedge \neg Rax)$

B. $\exists x\,(Qx \wedge \forall y\,(Qy \rightarrow y = x) \wedge \neg Rax)$

$P$: … is a table (piece of furniture)
$Q$: … is a table (with columns and rows)
$R$: … can find …

(iii) Fiona bought the same car as Philip.

*Answer.* Ambiguity between qualitative and numerical identity. It can be understood as numerical identity because two different persons can have bought the (numerically) same car at different times in the past.

A. $\exists x\,(Qx \wedge Rax \wedge Rbx)$

B. $\exists x\,\exists y\,(Qx \wedge Qy \wedge Rax \wedge Rbx \wedge Pxy)$

$a$: Fiona
$b$: Philip
$Q$: … is a car
$P$: … is qualitatively identical to …
$R$: … bought …

(iv) Paul ate the crisps in the kitchen.

*Answer.* It's unclear whether 'in the kitchen' refers to the crisps or to the place where Paul ate the crisps. There are different ways to formalise such sentences. Here is one possibility:

A. $\forall x\,(Px \wedge Qx \rightarrow Pax)$
for the reading 'Paul ate all the crisps that were in the kitchen'.

B. $\forall x\,(Px \rightarrow R^3_1axb$

$a$: Paul
$b$: the kitchen
$P$: … is a crisp
$Q$: … is in the kitchen
$R^2$: … ate …
$R^3_1$ … ate … in …

There are a few problematic issues here. For instance, 'the kitchen' is a definite description that should be be formalised as such.

(v) The data will be released and the chairman will resign if the allegations are true.

*Answer.* This is a scope ambiguity. Propositional logic is sufficient to bring out the two different readings.

A. $R \rightarrow (P \wedge Q)$

B. $P \wedge (R \rightarrow Q)$

$P$: The data will be released
$Q$: The chairman will resign
$R$: The allegations are true

(b) Give formalisations of the following sentences that are as detailed as possible. Explain for each formalisation why it is adequate and why a more detailed formalisation is not possible in the language of predicate logic with identity.

(i) Tom believes whatever Tim tells him.

*Answer.* One formalisation is $\forall x\,(Pabx \rightarrow Qbx)$.

$P$: … tells … …
$Q$: … believes …

(ii) It's conceptually true that bachelors are unmarried.

*Answer.* There is not much one can do about this as 'it's conceptually true that' isn't a truth-functional connective. So one can formalise the sentence as a single sentence letter.

Alternatively one could give the paraphrase 'The proposition that bachelors are unmarried is conceptually true.' and formalise the sentence accordingly as $Pa$.
$P$: … is conceptually true
$a$: the proposition that bachelors are unmarried

(iii) Water is a chemical element, and John believes that it has been found on Mars.

*Answer.* This is not easy. $Pa \wedge Qba$
$P$: … is a chemical element
$Q$: … believes of … that it is found on Mars

The justification for taking $Q$ as binary predicate letter is that the translation of $Q$ expresses a relation. If that's correct then John believes of water independently of how he describes it that it's found on Mars. So there isn't a problem with intentionality here.

There might be different views. And, yes, water isn't a chemical element, but that doesn't affect the formalisation.

(iv) George believes in God.

*Answer.* I think the best you can do is to formalise the sentence as $Pa$ where $P$ is translated as 'believes in God'.

(c) Is the set with the following English sentences as elements consistent or not? Justify your claim by providing formalisations of these sentences in the language of predicate logic with identity and by arguing for the consistency or inconsistency of this set.

> The lighthouse of Pharos does not exist anymore. The lighthouse of Pharos was tall and is still considered an outstanding achievement.

*Answer.* There are a couple of issues here, and I don't propose one particular answer here as there are so many different possible solutions. The issues that should be addressed include the following:

- Is 'the lighthouse of Pharos' a definite description or a proper name? Presumably it's a proper name because even if there had been another lighthouse in Pharos, there would still be *the* lighthouse of Pharos.
- How does one formalise 'does not exist (anymore)'? One could introduce a predicate letter for present existence and a predicate letter for past existence, or talk explicitly about periods of point in time.
- Formalisations of the first sentence as $\neg\exists x\, x = a$ ($a$ is translated as 'the lighthouse of Pharos') or the like will lead to an inconsistency in most cases. so existential quantification and actual existence are separated.