

SAMPLE PAPER

INTRODUCTION TO PHILOSOPHY

SECTION A: LOGIC

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1. (a) If a set of English sentences is propositionally consistent, is it then also consistent (simpliciter)? Substantiate your answer.
- (b) If the conclusion of an English argument is a tautology can the argument fail to be logically valid?
- (c) Show that the sentences in the following paragraph make up a propositionally inconsistent set by symbolizing them and by showing that the set of formalisations is semantically inconsistent (either by the truth table method or by a proof in Natural Deduction). Specify your dictionary carefully, and comment on anything difficult or otherwise noteworthy about the symbolization.

The planet Venus is visible in the evening—hence the use of a name ‘Hesperus’ (the Evening Star)—and it’s visible a second time in the morning—hence the use of a name ‘Phosphorus’ (the Morning Star). This implies that ‘Hesperus’ and ‘Phosphorus’ are different names for the very same planet, which in turn implies that the sentence ‘Hesperus is the same planet as Phosphorus’ is true. However, it’s possible—in fact, it’s no doubt actually the case—that there are people who are good enough logicians to know that ‘Hesperus is the same planet as Hesperus’ is true and yet don’t know that ‘Hesperus is the same planet as Phosphorus’ is true. Now, either ‘Hesperus is the same planet as Hesperus’ and ‘Hesperus is the same planet as Phosphorus’ mean the same thing or they don’t. If they do, then it must after all be the case that anyone who’s a good enough logician

to know that ‘Hesperus is the same planet as Hesperus’ is true will also know that ‘Hesperus is the same planet as Phosphorus’ is true. On the other hand, if these sentences mean different things, then ‘Hesperus’ and ‘Phosphorus’ will mean different things. But they’ll only mean different things if they’re names for different planets.

2. (a) Establish the following claims by means of proofs in the system of Natural Deduction:

$$(i) P \wedge Q \wedge R \vdash (P \rightarrow P_1) \rightarrow ((P_1 \rightarrow Q_1) \rightarrow Q_1)$$

$$(ii) Pa \wedge Qc, \forall x (Px \rightarrow Rxa) \vdash \exists y Ryy \vee \neg \exists z Qz$$

$$(iii) \forall x \exists y \neg Rxy \vdash \forall x \exists y \exists z (\neg Rxy \wedge \neg Ryz)$$

(b) Assume that ϕ and all elements of Γ are \mathcal{L}_2 -sentences. Using the completeness or soundness theorem for \mathcal{L}_2 show that $\Gamma \not\models \phi$ implies $\Gamma \not\vdash \phi$.

(c) Establish the following claims by means of counterexamples.

$$(i) P \wedge Q \wedge R \not\models (P \rightarrow P_1) \rightarrow (P_1 \rightarrow Q_1)$$

$$(ii) Pa \wedge Qc, \forall x (Px \rightarrow Rxa) \not\models \exists y Ryy \wedge \neg \exists z Qz$$

$$(iii) \forall x \exists y \neg Rxy \not\models \forall x \exists y \exists z (\neg Rxy \wedge \neg Ryz \wedge \neg Rzx)$$

3. (a) Formalise the following sentences in predicate logic using the following dictionary noting any difficulties of points of interest:

P : ... is a pen

Q : ... owns ...

R : ... is red

P_1 : ... is green

a : John

(i) Some pens are red, some are not.

(ii) John owns a pen that isn’t red, but he also owns a red pen.

(iii) John owns only red or green pens.

(iv) Unless one of Johns pens is red, all of his pens are green.

(v) John owns something green or red which isn’t a pen.

(b) Show that the following argument is valid in predicate logic noting any difficulties or points of interest.

Some book authors haven’t written a novel. Therefore there are books that aren’t novels.

4. (a) Explain why the following attempted proofs are not correct proofs in the system of Natural Deduction. Note all steps that are not correct. Give complete correct proofs for each of the claims.

(i) $P \rightarrow Q \vdash \neg P \vee Q$

$$\frac{\frac{[P] \quad P \rightarrow Q}{Q}}{\frac{\neg P \vee Q}{\neg P \vee Q}} \quad \frac{[\neg(\neg P \vee Q)]}{\frac{\neg P}{\neg P \vee Q}}$$

(ii) $\exists x Q_2xx \vdash \exists z Q_2zz$

$$\frac{\frac{\exists x Q_2xx}{Q_2aa}}{\exists z Q_2zz}$$

(iii) $\forall x \exists y (Qxy \wedge Pyx) \vdash \exists x \exists y \exists z (Qxy \wedge Pzy)$

$$\frac{\frac{\frac{\forall x \exists y (Qxy \wedge Pyx)}{\exists y (Qay \wedge Pya)} \quad \frac{\frac{\forall x \exists y (Qxy \wedge Pyx)}{\exists y (Qby \wedge Pyb)} \quad \frac{\frac{[Qab \wedge Pba] \quad [Qbc \wedge Pcb]}{Qab \quad Pcb}}{Qab \wedge Pcb}}{\exists x (Qxb \wedge Pcb)}}{\exists x \exists y (Qxy \wedge Pcy)}}{\exists x \exists y \exists z (Qxy \wedge Pzy)} \exists\text{Elim}$$

- (b) State the rule \neg -Intro for the introduction of \neg . If there is a proof of ϕ from undischarged assumptions in Γ in the system of Natural Deduction then there is also a proof of ϕ from undischarged assumptions in Γ without an application of \neg -Intro. Show why \neg -Intro is dispensable.

5. (a) Determine for each of the following relations

- whether it is reflexive on the set of all \mathcal{L}_1 -sentences,
- whether it is symmetric,
- whether it is antisymmetric,
- whether it is asymmetric, and
- whether it is transitive.

Substantiate your answers. In the following ϕ and ψ are understood to be \mathcal{L}_1 -sentences.

(i) The set of all pairs $\langle \phi, \psi \rangle$ such that $\phi \vDash \psi$

- (ii) The set of all pairs $\langle \phi, \psi \rangle$ such that $\phi \models \neg\psi$
 - (iii) The set of all pairs $\langle \phi, \psi \rangle$ such that $\phi \wedge \neg\phi \models \psi$
- (b) What is a function? Determine for each of the following relations whether it is a function or not. Substantiate your answers.
- (i) The set of all pairs $\langle d, e \rangle$ such that d is a person and e is d 's head
 - (ii) The set of all pairs $\langle e, d \rangle$ such that d is a person and e is d 's head
 - (iii) The set of all pairs $\langle d, e \rangle$ such that d is a person and e is one of e 's toes
 - (iv) The set of all pairs $\langle e, d \rangle$ such that d is a person and e is one of e 's toes
- (c) Answer each of the following questions and give a reason for your answer either by providing an example having the properties in question or by proving that such a relation cannot exist.
- (i) Is there a relation that is symmetric and asymmetric?
 - (ii) Is there a function that is reflexive on the set of all Oxford colleges and that is not transitive if all components of ordered pairs in the relation are Oxford colleges?
 - (iii) Is there a relation containing more than two ordered pairs that is asymmetric, transitive, and a function?
6. (a) How is an \mathcal{L}_2 -structure defined?
- (b) Provide counterexamples that establish the following claims. You don't have to prove that the premisses are true in the \mathcal{L}_2 -structure and that the conclusion is false.
- (i) $\forall x Rxx \neq \exists x \forall y Rxy$
 - (ii) $\exists x (Px \wedge \exists y Rxy), \forall x (Px \rightarrow \neg \exists z (Rxz \wedge Pz)) \neq Q$
- (c) Consider an \mathcal{L}_2 -structure \mathcal{S} with the domain $D_{\mathcal{S}}$ and the following semantic values of a and R :

$$D_{\mathcal{S}} = \{\text{Europe, Asia, Australia}\}$$

$$|a|_{\mathcal{S}} = \text{Europe}$$

$$|R|_{\mathcal{S}} = \{\langle \text{Australia, Europe} \rangle, \langle \text{Europe, Asia} \rangle, \langle \text{Australia, Asia} \rangle\}$$

Are the following sentences true or false in this structure? Justify your answers as fully as possible.

- (i) $\exists x Rax$
- (ii) $\forall x Rax \rightarrow (\neg Raa \vee \forall x \forall y Rxy)$
- (iii) $\forall x \exists y Rxy$

(iv) $\exists x Rxx$

7. (a) For each of the sentences below explain the way in which it is ambiguous. If possible, reveal the ambiguity by formalising the sentence in two (or more) different ways using the same dictionary in each of its formalisations.
- (i) Some oak species can be found on every continent.
 - (ii) Tom can't find the table.
 - (iii) Fiona bought the same car as Philip.
 - (iv) Paul ate the crisps in the kitchen.
 - (v) The data will be released and the chairman will resign if the allegations are true.
- (b) Give formalisations of the following sentences that are as detailed as possible. Explain for each formalisation why it is adequate and why a more detailed formalisation is not possible in the language of predicate logic with identity.
- (i) Tom believes whatever Tim tells him.
 - (ii) It's conceptually true that bachelors are unmarried.
 - (iii) Water is a chemical element, and John believes that it has been found on Mars.
 - (iv) George believes in God.
- (c) Is the set with the following English sentences as elements consistent or not? Justify your claim by providing formalisations of these sentences in the language of predicate logic with identity and by arguing for the consistency or inconsistency of this set.

The lighthouse of Pharos does not exist anymore. The lighthouse of Pharos was tall and is still considered an outstanding achievement.