The solutions are highly incomplete and only intended to give a rough idea.

1. (a) Which of the following expressions is an abbreviation of a sentence of $\mathscr{L}_1$? If an expression is an abbreviation of a $\mathscr{L}_1$-sentence, then restore all the brackets that have been dropped in accordance with the Bracketing Conventions of the Logic Manual. If not, explain why not. [4]

   (i) $P \vee Q \vee R \leftrightarrow \neg(\neg P \wedge \neg Q \wedge \neg R)$

   (ii) $P \vee Q \wedge R \rightarrow P \vee (Q \rightarrow R)$

   (b) What is it for two $\mathscr{L}_1$-sentences to be logically equivalent? [1]

   (c) Which of the following are pairs of logically equivalent $\mathscr{L}_1$-sentences? Use truth tables in order to justify your answer. [4]

   (i) $(P \rightarrow Q) \wedge \neg Q,\ P \rightarrow Q \wedge \neg Q$

   (ii) $P \rightarrow (Q \rightarrow R),\ R \rightarrow (\neg Q \rightarrow \neg P)$

   (d) Show that the following argument can be turned into a propositionally valid argument by appropriately rewording its premises and adding any further assumptions upon which the speaker might naturally be expected to be relying if required. Justify your answer by means of a partial truth table or a proof in Natural Deduction. Specify your dictionary carefully and note any difficulties or points of interest. [16]

   I just raised my hand. Either determinism is false or the laws of nature being what they in fact are and the events in the remote past being what they in fact are entailed my raising my hand just now. But if the laws of nature being what they are together with the events in the remote past being what they are entailed my raising my hand, then I only had any control over my doing so if I had any control over the laws of nature being what they in fact are or the events in the remote past being what they in fact are. It is plain that I had no control whatever over the laws of nature being what they in fact are. Nor did I had any control over the events in the remote past being what they in fact are. It follows that I had no control whatever over my raising my hand. For determinism is true after all.

2. (a) Show that each of the following arguments is in fact propositionally valid. You may use a partial truth table or a Natural Deduction proof to show that an argument is propositionally valid. [9]

   (i) If Brown decides to visit Spain, then he will not visit Barcelona. Therefore, if Brown visits Barcelona, then Brown will not decide to visit Spain.

   (ii) It is not the case that, if God exists, then human life is the product of random chance. So, God exists.

   (iii) If I turn the ignition on, then the engine will start. Therefore, if I turn the ignition on and there is no petrol in the tank, then the engine will start.

   (b) Write a brief essay discussing the question of whether we should conclude that $\rightarrow$ is inadequate for the formalisation of ‘if ... then ...’ in English or whether we could still argue that, despite appearances to the contrary, $\rightarrow$ provides an appropriate translation for the use of ‘if ... then ...’ in the previous three arguments. [16]

3. (a) Write an essay comparing and contrasting the following two rival characterisations of logical validity. Are there any reasons to prefer one over the other? [13]

   (A) *An argument is logically valid if and only if there is no interpretation under which the premises are true and the conclusion is false*.

   (B) *An argument is logically valid if and only if it is impossible for the premises to be true while the conclusion is false*.

   (b) Which of the following arguments are logically valid? If an argument has a valid formalisation in $\mathscr{L}_1$, $\mathscr{L}_2$ or $\mathscr{L}_=$, then specify your dictionary and use a partial truth table in the case of arguments in $\mathscr{L}_1$ or a Natural Deduction proof in the case of arguments in $\mathscr{L}_2$ or $\mathscr{L}_=$ to show the validity of the resulting argument. If an argument cannot be formalised into a valid argument in $\mathscr{L}_=$, then specify your dictionary and provide a counterexample. (There is no need to prove that your structure is a counterexample; you need only specify the structure and briefly sketch the reasons why it counts as a counterexample.) Finally, explain whether your formalisation is adequate. [12]

   (i) Some apples are red all over and some apples are green all over. As a consequence, some apples are both red and green all over.

   (ii) Nothing is both red and green all over.

(iii) Most tutors hold their tutorials in the afternoon. Most tutors hold their tutorials in College. Therefore, some tutors must hold their tutorials in College in the afternoon.

(iv) If God is perfectly good, then there is evil in the world only if God cannot prevent it. If there is evil in the world, and God cannot prevent it, then God is not omnipotent. But there is evil in the world. Therefore, God is not both omnipotent and perfectly good.

4. (a) Establish each of the following by means of a proof in the system of Natural Deduction:
[12]

(i) $P_1 \vee P_2 \to Q \vee R, \neg(P_1 \to Q) \vdash P_2 \to R$

(ii) $\forall x(Px \to (Q_1x \vee Q_2x)), \neg\exists x(Q_1x \wedge Rx), \exists x(Px \wedge Rx) \vdash \exists x Q_2x$

(iii) $\exists x Rxx, \forall x \forall y(Rxx \wedge Rxy \to x = y) \vdash \exists x \forall y(Rxy \leftrightarrow x = y)$

(b) Find any mistakes in the following attempted proofs. List all steps in the proof that are not licensed by a rule of Natural Deduction. If there is a repair, supply a correct proof. If not, show that the argument is not valid by means of a counterexample. (There is no need to prove that your structure is a counterexample; you need only specify the structure and briefly sketch the reasons why it counts as a counterexample.)
[13]

(i) $R, \neg(P \wedge Q \wedge R), \neg P \vee \neg R \to P_1 \wedge P_2 \vdash Q \to P_1 \wedge P_2$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{R \quad [Q]}{Q \wedge R} \quad [P]}{P \wedge Q \wedge R} \quad \neg(P \wedge Q \wedge R)}{\neg P}}{\neg P \vee \neg R} \quad \neg P \vee \neg R \to P_1 \wedge P_2}{P_1 \wedge P_2}}{Q \to P_1 \wedge P_2}$$

(ii) $\vdash \neg(\exists x Px \to \forall x Px) \to (\exists x \neg Px \to \exists x Px)$

$$\dfrac{\exists x \neg Px \qquad \dfrac{\dfrac{\neg Pa \quad \dfrac{[\forall x Px]}{Pa}}{\neg\forall x Px} \qquad \dfrac{\neg\exists x Px \to \forall x Px \quad [\neg\exists x Px]}{\forall x Px}}{\exists x Px}}{\exists x Px}$$

(iii) $\exists x Rxx, \forall x \forall y(Rxy \land Ryy \to x = y) \vdash \exists x \forall y(Ryy \to x = y)$

$$\dfrac{\exists x Rxx \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{Raa}{Raa \land Raa} \qquad \dfrac{\dfrac{\forall x \forall y(Rxy \land Ryy \to x = y)}{\forall y(Ray \land Ryy \to a = y)}}{Raa \land Raa \to a = a}}{a = a}}{Raa \to a = a}}{\forall y(Ryy \to a = y)}}{\exists x \forall y\,(Ryy \to x = y)}}{\exists x \forall y\,(Ryy \to x = y)}$$

5. (a) Use the following dictionary to translate each of the following sentences of $\mathscr{L}_2$ into idiomatic English: [7]

P: ... is a critic
Q: ... is knowledgeable
R: ... admires ...

(1) $\forall x\,(\exists y\,(Rxy \land Py) \to Px)$

(2) $\forall x\,(Px \to Qx)$

(3) $\exists x\,(Qx \land \forall y\,\neg(Rxy \land Py))$

(4) $\exists x\,Rxx$

(5) $\exists x\,\neg Rxx$

(b) Find any $\mathscr{L}_2$-structures satisfying each of the following conditions. There is no need to prove that your structure satisfies the relevant condition; you need only specify the structure and briefly sketch the reasons why your proposed structure satisfies the condition. If there is no structure satisfying the relevant condition, then justify your answer. [18]

(i) Each of (1), (3) and (5) in part (a) are true and (2) and (4) are false.

(ii) Each of (1), (2) and (4) in part (a) are true and (3) and (5) are false.

(iii) Each of (2), (3) and (4) in part (a) are true and (1) and (5) are false.

(iv) Each of (2), (4) and (5) in part (a) are true and (1) and (3) are false.

(v) Every $\mathscr{L}_2$-sentence in part (a) is true.

(vi) Every $\mathscr{L}_2$-sentence in part (a) is false.

6. (a) Add quotation marks to the following sentences in order to obtain true non-ambiguous English sentences whenever possible. Comment on any points of interest and specify whether you think there is more than one way—or none—to answer the question. [8]

   (i) α, β, and γ are Greek letters.

   (ii) Paris is identical to London is a sentence.

   (iii) and are quotation marks.

   (iv) This is the first word of this incomplete sentence.

   (b) Define the notion of a sentence of the language $\mathscr{L}_1$ of propositional logic. [5]

   (c) The negation of a $\mathscr{L}_1$-sentence is the sentence preceded by the symbol '¬'. Determine for each of the following relations whether it is reflexive on the set of $\mathscr{L}_1$-sentences, whether it is symmetric on this set and whether it is transitive on it. [6]

   (i) the set of all ordered pairs $\langle e, f \rangle$ where f is the negation of e

   (ii) the set of all ordered pairs $\langle e, f \rangle$ where f is the negation of e or e is the negation of f

   (iii) the set of all ordered pairs $\langle e, f \rangle$ where the set $\{e, f\}$ is semantically consistent.

   (d) For each of the sentences below, explain the ways in which it is ambiguous. Whenever possible, reveal the ambiguity by formalising each sentence in two (or more) different ways in $\mathscr{L}_2$. Specify your dictionary carefully and note any points of interest. [6]

   (i) All that glitters is not gold.

   (ii) There is a solution to every problem.

   (iii) No one is allowed to have ice cream and cake.

7. (a) Formalise the following sentences in $\mathscr{L}_=$ using the following dictionary: [8]

   P: ... is an English county
   Q: ... is in the South West
   R: ... is adjacent to ...
   a: Oxfordshire
   b: Berkshire
   c: Cumbria

   (i) Cumbria is an English county which is not adjacent to Oxfordshire or Berkshire.

(ii) No English county is adjacent both to Cumbria and to Oxfordshire, though some English counties are adjacent both to Oxfordshire and Berkshire.

(iii) Every English county has at least two adjacent counties.

(iv) One English county in the South West has only one adjacent county.

(b) Consider an $\mathscr{L}_=$-structure $\mathscr{S}$ satisfying the following conditions:

$$D_{\mathscr{S}} = \{x : x \text{ is an English county}\}$$
$$|P|_{\mathscr{S}} = \{x : x \text{ is in the South East}\}$$
$$|R|_{\mathscr{S}} = \{\langle e, d\rangle : e \text{ is adjacent to } d\}$$

Let α be a variable assignment with:

$$|x|^{\alpha}_{\mathscr{S}} = \text{Oxfordshire}$$
$$|y|^{\alpha}_{\mathscr{S}} = \text{Berkshire}$$
$$|z|^{\alpha}_{\mathscr{S}} = \text{Cumbria}$$

Does α satisfy any of the following formulae in $\mathscr{S}$? Justify your answer. (You do not have to give a complete proof, but you should say why you think each formula is or not satisfied in $\mathscr{S}$ by the assignment.) [9]

(i) $\exists x \forall y Rxy$

(ii) $\forall x \exists y Rxy \to Px$

(iii) $\forall x (\exists y Rxy \to Px)$

(c) For each of the sentences below, explain the ways in which it is ambiguous. Whenever possible, reveal the ambiguity by formalising each sentence in two (or more) different ways in $\mathscr{L}_=$. Make sure to specify your dictionary carefully and note any points of interest. [8]

(i) No two people own the same car.

(ii) The man on the moon is not an astronaut.