I thank the author of the examination paper on which sample paper is based. VH

1. (a) Which of the following expressions is a sentence of $\mathscr{L}_1$ or an abbreviation of one? If an expression is neither a sentence of $\mathscr{L}_1$ nor an abbreviation of one, then explain why not. If it is an abbreviation of a sentence of $\mathscr{L}_1$, then restore all the brackets that have been dropped in accordance with the Bracketing Conventions of the Logic Manual. [3]

   (i) $((\neg P \to (Q \lor R)) \leftrightarrow T)$

   (ii) $\neg P \neg \to (Q_1 \lor Q_2)$

   (iii) $P \land Q \leftrightarrow Q \lor P$

   (b) Determine whether each of the following sentences of $\mathscr{L}_1$ is a tautology or a contradiction or neither. Use truth tables in order to justify your answer. [6]

   (i) $((P \lor (Q \land R)) \leftrightarrow ((P \lor Q) \land (P \lor R)))$

   (ii) $(\neg R \to ((Q \land \neg P) \leftrightarrow (R \lor (P \to Q))))$

   (iii) $(\neg(P \land Q) \leftrightarrow (\neg P \lor \neg Q))$

   (c) Show that the following argument can be turned into a propositionally valid argument by appropriately rewording its premises and adding any further assumptions upon which the speaker might naturally be expected to be relying if required. Justify your answer by means of a partial truth table or a proof in Natural Deduction. Specify your dictionary carefully and note any difficulties or points of interest. [16]

   > The Russians will dismantle their Cuban missile bases if, but only if, the Americans carry out their threat. If Castro fails to stay power, then the Russians will dismantle their Cuban missiles. But the Americans will carry out their threat only if Kennedy stays in office and Krushev fails to agree to their demands. Although Kennedy did indeed stay in office, Kruschev finally agreed to the American demands. Therefore, Castro must have stayed in power.

2. (a) Determine whether each of the following arguments is propositionally valid. You may use a partial truth table or a Natural Deduction proof to show that an argument is propositionally valid, but you must supply an interpretation to show that an argument is propositionally invalid. [12]

   (i) If the followers of Averroes are right, then the world is eternal only if it has never been created. But if Aquinas is right, then it is not true that the world has been created only if it is not eternal. But either the world is eternal or it is not. Therefore, if the world has been created and the followers of Averroes are right, then Aquinas is wrong.

   (ii) If Tom is not studying the Logic Manual, then he is attending a logic lecture. And if Tom is not attending a logic lecture, then he is doing some exercises. Tom cannot be doing more than one of these activities now. Therefore, Tom must now be attending a logic lecture.

(b) Determine whether each of the following arguments is valid in predicate logic. You should use a Natural Deduction proof to show that an argument is valid in predicate logic, and you must supply a counterexample to show that an argument is not valid in predicate logic. [13]

   (i) Descartes was a philosopher who was also a mathematician. Not all philosophers have an interest in set theory, but all mathematicians do. Therefore, some philosophers have an interest in set theory.

   (ii) Othello loves Desdemona. Iago despises Cassio. Everyone Iago despises loves Desdemona. Therefore, Iago despises Othello and Cassio loves Desdemona.

3. (a) Establish each of the following by means of a proof in the system of Natural Deduction: [12]

   (i) $\neg P \to Q,\ R \vee \neg Q,\ P \to (Q_1 \vee Q_2),\ \neg R \wedge \neg Q_2 \vdash Q_1$

   (ii) $\exists x(Px \wedge Qx \wedge Rax),\ Pb \wedge Qb,\ \forall x \forall y(Px \wedge Qx \wedge Py \wedge Qy \to x = y) \vdash Rab$

   (iii) $\forall x \forall y(Rxy \to Ryx),\ \forall x \forall y \forall z(Rxy \wedge Ryz \to Rxz),\ \forall x \exists y Rxy \vdash \forall x Rxx$

   (b) Find any mistakes in the following attempted proofs. List all steps in the proof that are not licensed by a rule of Natural Deduction. If there is a repair, supply a correct proof. If not, show that the argument is not valid by means of a counterexample. (There is no need to prove that your structure is a counterexample; you need only specify the structure and briefly sketch the reasons why it counts as a counterexample.) [13]

   (i) $P \leftrightarrow Q,\ P_1 \wedge P_2,\ \neg P \vdash \neg Q \wedge P_2$

$$\dfrac{\dfrac{\dfrac{[Q \wedge P_2]}{Q} \qquad P \leftrightarrow Q}{P} \qquad \neg P}{\neg Q \wedge P_2}$$

   (ii) $\forall x \exists y Rxy \to \neg \exists x Rxx,\ \exists x \forall y Ryx \vdash \exists x \neg Rxx$

$$\dfrac{\dfrac{\dfrac{[Raa]}{\exists x Rxx} \qquad \dfrac{[\exists x \forall y Rxy] \qquad \forall x \exists y Rxy \to \neg \exists x Rxx}{\neg \exists x Rxx}}{\neg Raa}}{\exists x \neg Rxx}$$

   (iii) $\forall x(Px \to Rax),\ \forall x(Rxx \to Qx),\ \exists x Qx \to \forall y P_1 y \vdash \forall x(Px \to P_1 x)$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[Pa] \qquad \dfrac{\forall x(Px \to Rax)}{Pa \to Raa}}{Raa} \qquad \dfrac{\forall x(Rxx \to Qx)}{Raa \to Qa}}{Qa}}{\exists x Qx} \qquad \exists x Qx \to \forall y P_1 y}{\forall y P_1 y}}{P_1 a}}{Pa \to P_1 a}}{\forall x(Px \to P_1 x)}$$

4. (a) Formalise the following sentences in the language $\mathscr{L}_=$ using the following dictionary:

R: … knows …
a: Iago
b: Othello
c: Desdemona

[14]

(i) Everyone knows Othello.

(ii) Someone knows everyone.

(iii) Othello knows Desdemona

(iv) Everyone knows someone who doesn't know him.

(v) There is someone who knows everyone who knows him

(vi) No one who knows Othello knows anyone who knows Iago.

(vii) Someone is such that he knows Othello only if everyone does.

(b) Determine whether the following argument is valid in predicate logic with identity. You should use a Natural Deduction proof to show that the argument is valid in predicate logic with identity or a counterexample to show that the argument is not valid in predicate logic with identity. [5]

(i) Everyone knows everyone who knows someone. Desdemona knows Iago. Therefore, Iago knows Othello.

(c) For each of the sentences below, explain all the ways in which it is ambiguous. Whenever possible, reveal the ambiguity by formalising each sentence in two (or more) different ways in $\mathscr{L}_=$. Specify your dictionary carefully and note any points of interest. [6]

(i) Tom learned something from each of his tutors.

(ii) God helps those who help themselves.

(iii) There are games only two people can play.

5. (a) Add quotation marks to the following sentences in order to obtain true non-ambiguous English sentences whenever possible. Comment on any points of interest and specify whether you think there is more than one way—or none—to answer the question. [8]

   (i) In Spanish, la and nieve and es and blanca may be combined to form la nieve es blanca.

   (ii) Oxford is to the west of London but London is not to the east of Oxford.

   (iii) The last word of the best solution for (ii) is Oxford.

   (iv) The first letter of the Greek alphabet is α is a true sentence.

(b) For each of the sentences below, explain all the ways in which it is ambiguous. Whenever possible, reveal the ambiguity by formalising each sentence in two (or more) different ways in $\mathscr{L}_=$. Specify your dictionary carefully and note any points of interest. [4]

   (i) A person who has a Ferrari is admired by a person without one.

   (ii) People like others if they like themselves.

(c) What is the scope of an occurrence of connective in a sentence of the language $\mathscr{L}_1$ of propositional logic? [2]

(d) Determine the scopes of the underlined occurrences of quantifiers after adding any brackets that have been omitted in accordance with the rules for saving brackets. [3]

   (i) $P \wedge Q \leftrightarrow R_1 \underline{\vee} R_2$

   (ii) $P \vee Q \underline{\vee} R \leftrightarrow \underline{\neg}(\neg P \wedge \neg Q \wedge \neg R)$

   (iii) $P \vee Q \vee \underline{\neg} R \vee P_1 \underline{\rightarrow} Q$

(e) (i) Give an example of a truth-functional connective in English other than the standard connectives—‘and’, ‘or’, ‘it is not the case that’, ‘if ... then’ and ‘if and only if’. Explain what makes it truth-functional. [8]

   (ii) Give an example of a non-truth-functional connective in English, and explain why it is not truth-functional.

6. (a) Determine for each of the following relations whether it is reflexive on the set of $\mathscr{L}_1$-sentences, whether it is symmetric on this set and whether it is transitive on it. [6]

   (i) the set of all ordered pairs $\langle e, f\rangle$ where e is logically equivalent to f.

   (ii) the set of all ordered pairs $\langle e, f\rangle$ where the set $\{e, f\}$ is semantically inconsistent.

   (iii) the set of all ordered pairs $\langle e, f\rangle$ where the set $\{e, f\}$ is semantically consistent.

(b) Consider an $\mathscr{L}_2$-structure $\mathscr{S}$ satisfying the following conditions:

$$\begin{aligned} D_{\mathscr{S}} &= \{1,2,3,4,5\} \\ |P|_{\mathscr{S}} &= \{1,2,3\} \\ |Q|_{\mathscr{S}} &= \{3,5\} \\ |R_1|_{\mathscr{S}} &= \{\langle 1,2\rangle, \langle 1,3\rangle, \langle 2,3\rangle, \langle 3,3\rangle\} \\ |R_2|_{\mathscr{S}} &= \{\langle 1,2\rangle, \langle 3,1\rangle, \langle 3,3\rangle\} \end{aligned}$$

Find a variable assignment α over $\mathscr{S}$ satisfying each of the following formulas in $\mathscr{S}$. Justify your answer. (You do not have to give a complete proof, but you should say why you think each formula is satisfied in $\mathscr{S}$ by the assignment.) [12]

   (i) $(Px \lor Qx)$

   (ii) $(Px \leftrightarrow Qx)$

   (iii) $(\neg Px \lor \neg Qx)$

   (iv) $\exists x\, (R_1xy \land R_2xy)$

   (v) $\forall x(\exists y R_1xy \to R_1xz)$

   (vi) $\forall x((Px \land Qx) \to \neg R_1xy)$

(c) (i) Provide a sentence of the language of $\mathscr{L}_2$ that is true in some structures with a domain containing at least two elements but that is not true in any structure containing only one element. [7]

   (ii) Provide a sentence of the language of $\mathscr{L}_=$ that is true in all and only structures with a domain consisting of exactly two elements.

   (iii) Provide a sentence of the language of $\mathscr{L}_=$ that is true in all and only structures with a domain consisting of at most three elements.

7. (a) What is for an argument in English to be logically valid? [1]

   (b) What is for an argument in English to be propositionally valid? [1]

   (c) What is for an argument in English to be valid in predicate logic? [1]

   (d) True or false. Justify your answer. [12]

      (i) Every logically valid argument in English is either propositionally valid or valid in predicate logic with identity.

      (ii) Every propositionally valid argument is valid in predicate logic.

      (iii) If an argument is valid in predicate logic with identity, then it is valid in predicate logic.

      (iv) Some logically valid arguments are not propositionally valid.

   (e) Are the following arguments valid? Are they propositionally valid? (You may use a partial truth table or a Natural Deduction proof to show that an argument is propositionally valid.) Are they valid in predicate logic? Are they valid in predicate logic with identity? Explain your answers making sure you discuss any points of interest. [10]

      (i) Anyone who ponders the question of why there is something rather than nothing is a metaphysician. A person ponders the question of why there is something rather than nothing if and only if she finds it interesting. So, anyone who finds the question of why there is something rather than nothing interesting is a metaphysician.

      (ii) Either logic is hard or it is not a popular subject but not both. But if macroeconomics is a popular subject, then so is logic. So, logic is hard only if macroeconomics is not a popular subject.

      (iii) Nothing is both red and green all over. Therefore, there is something rather than nothing.

      (iv) Russell and Whitehead wrote *Principia Mathematica*. *Principia Mathematica* is a three-volume book. Therefore, Russell wrote a three-volume book.