Collection paper

INTRODUCTION TO LOGIC

Hilary Term 2009

TIME:

Please answer questions.

1. (a) What does it mean for an argument in English to be propositionally valid? 2

   (b) If the conclusion of an English argument is a tautology, can the argument be valid? And can it be not valid? 2

   (c) What is the scope of an occurrence of connective in a sentence of the language $\mathcal{L}_1$ of propositional logic? 2

   (d) Determine the scopes of the underlined occurrences of quantifiers after adding any brackets that have been omitted in accordance with the rules for saving brackets. 3

   (i) $P \to Q \vee R_{23} \underline{\vee} R_{23}$

   (ii) $\neg\underline{\neg}(P \underline{\leftrightarrow} Q \wedge P_3) \vee (P_2 \wedge \neg R)$

   (e) Show that the following argument can be transformed into a propositionally valid argument if the premisses are appropriately reformulated and if premisses are added on which one may naturally rely. You may use the truth table method or give a proof in Natural Deduction. Specify your dictionary carefully and note any difficulties or points of interest. formalisation: 8, truth table: 4, comments: 4

   > If Brown sold his old Ford, he must have used the money for a trip to the Caribbean and he must be there now. Otherwise he could only have afforded a trip to Barcelona and he must be there by now. Nobody else would buy the beaten-up banger from Brown; only Jones could have bought that old Ford. So either Jones bought Brown's Ford or Brown is in Barcelona but it can't be the case that both are true, that is, that Jones bought the Ford and Brown is in Barcelona.

2. (a) Establish each of the following claims by means of proofs in the system of Natural Deduction:

   (i) $P \to (Q_1 \lor Q_2), \neg Q_1 \land \neg Q_2 \vdash \neg P$ 5

   (ii) $\forall x\, Qx \lor \forall x\, Rx \vdash \forall x\, (Qx \lor Rx)$ 5

   (iii) $\exists x\, (Px \to \forall y\, Ryy) \vdash \forall x\, Px \to Raa$ 5

(b) Explain why the following attempted proofs are not correct proofs in the system of Natural Deduction. Note all steps that are not correct. Give complete correct proofs for any true claims below and counterexamples to any false claims you find. 2

   (i) $P \lor Q, P \leftrightarrow Q \vdash P \land Q$

$$\dfrac{P \lor Q \qquad \dfrac{[P] \quad [Q]}{P \land Q} \qquad \dfrac{[P] \quad [Q]}{P \land Q}}{P \land Q}$$

   (ii) $\forall x\, (Px \lor Qx) \vdash \forall x\, Px \lor \forall x\, Qx$ 4

$$\dfrac{\dfrac{\forall x\, (Px \lor Qx)}{Pa \lor Qa} \qquad \dfrac{\dfrac{[Pa]}{\forall x\, Px}}{\forall x\, Px \lor \forall x\, Qx} \qquad \dfrac{\dfrac{[Qa]}{\forall x\, Qx}}{\forall x\, Px \lor \forall x\, Qx}}{\forall x\, Px \lor \forall x\, Qx}$$

   (iii) $\forall z_1\, \forall z_2\, (Rz_1z_2 \to Qz_2z_1), \exists x\, \exists y\, Rxy \vdash \exists z\, \exists y\, Qzy$ 4

$$\dfrac{\exists x\, \exists y\, Rxy \qquad \dfrac{\dfrac{[\exists y\, Rby] \qquad \dfrac{\dfrac{[Rba] \qquad \dfrac{\forall z_1\, \forall z_2\, (Rz_1z_2 \to Qz_2z_1)}{Rba \to Qab}}{Qab}}{\exists y\, Qay}}{\exists y\, Qay}}{\exists z\, \exists y\, Qzy}}{\exists z\, \exists y\, Qzy}$$

3. (a) Add quotation marks to the following expressions so that true and non-ambiguous English sentences are obtained if possible. Comment on any difficulties and indicate if there is more than one way to answer the question.

(i) The quotation of the quotation of ! is !. 2

(ii) $(P \rightarrow \neg Q)$ is a sentence of the language of propositional logic. 2

(iii) is an opening quotation mark. 2

(iv) It's raining and it's snowing and it's cold are English sentences. 2

(b) Consider the relation Q having all ordered pairs $\langle d, e\rangle$ as elements where d is the quotation of e. Answer the following questions and substantiate your answers:

(i) Is Q reflexive on the set of all strings of English expressions? 2

(ii) Is Q transitive? 2

(iii) Is Q symmetric? Is Q antisymmetric? Is Q asymmetric? 3

(c) The language $\mathcal{L}_{\mathrm{A}}$ is defined as follows (in the definition I have dropped quotation marks in accordance with the usual convention):

The letters A and B are sentences of $\mathcal{L}_{\mathrm{A}}$.
If ϕ and ψ are sentences of $\mathcal{L}_{\mathrm{A}}$, then $N\phi$ and $(I\phi\psi)$ are sentences of $\mathcal{L}_{\mathrm{A}}$.
Nothing else is a sentence of $\mathcal{L}_{\mathrm{A}}$.

(i) What are the Greek letters ϕ and ψ called if used as above? What is their use? 2

(ii) Is $(I(IAA)A)$ a sentence of $\mathcal{L}_{\mathrm{A}}$? Substantiate your answer. 2

(iii) Is the expression $N(INNANNB)$ a sentence of $\mathcal{L}_{\mathrm{A}}$? Substantiate your answer. 2

(iv) State rules for saving brackets in such a way that every abbreviation of an $\mathcal{L}_{\mathrm{A}}$-sentence abbreviates at most one $\mathcal{L}_{\mathrm{A}}$-sentence. Try to state a rule or rules that allow one to save as many brackets as possible. Explain why abbreviations do not abbreviate more than one sentence. 4

4. (a) Formalise the following sentences in the language $\mathcal{L}_=$ of predicate logic with identity using the following dictionary:

   P: … is a book
   Q: … is on Bill's desk
   R: … is red

   (i) The book on Bill's desk is red. 2

   (ii) Something on Bill's desk isn't red. 2

   (iii) There is something on Bill's desk that isn't a book. 2

   (iv) There are at least three books on Bill's desk. 2

   (v) There are at most two red things. 2

   (b) Show that the English argument with (i) and (ii) as premises and (iii) as conclusion is valid in predicate logic with identity. 10

   (c) Show that the formalisation of the argument with (i) and (iii) as premisses and (ii) as conclusion is not valid in predicate logic with identity by providing a counterexample. 5

5. (a) Show that the following argument is propositionally valid if it is suitably formalised. Note any points of interest. formalisation: 9 truth table: 4

   If ϕ implies ψ and ψ implies χ then ϕ and χ share some sentence letter, unless ϕ is a contradiction or χ is logically true. Therefore, ϕ is a contradiction or ϕ doesn't imply ψ or ψ doesn't imply χ, if ϕ and χ don't share a sentence letter, provided that χ is not logically true.

   (b) Determine for each of the following relations
   - whether it is reflexive on the set of all $\mathcal{L}_2$-sentences,
   - whether it is symmetric,
   - whether it is antisymmetric,
   - whether it is asymmetric, and
   - whether it is transitive.

   Substantiate your answers. In the following ϕ and ψ are understood to be $\mathcal{L}_1$-sentences.

   (i) The set of all pairs $\langle \phi, \psi \rangle$ such that $\phi \to \psi$ is a contradiction. 4

   (ii) The set of all pairs $\langle \phi, \psi \rangle$ such that $\phi \to \psi$ is a logical truth. 4

   (iii) The set of all pairs $\langle \phi, \psi \rangle$ such that for some $\mathcal{L}_2$-logical truth χ, $\psi \wedge \phi \vDash \chi$. 2

   (iv) The set of all pairs $\langle \phi, \psi \rangle$ such that for every $\mathcal{L}_2$-logical truth χ, $\psi \wedge \phi \vDash \chi$. 2

6. (a) How is an $\mathcal{L}_2$-structure defined? 2

(b) What are the semantic values (extensions) of constants in an $\mathcal{L}_2$-structure? 1

(c) What is a variable assignment over an $\mathcal{L}_2$-structure? 2

(d) Consider the following $\mathcal{L}_2$-structure $\mathcal{S}$:

$$
\begin{aligned}
D_{\mathcal{S}} &= \{d : d \text{ is a planet of the solar system}\} \\
|P|_{\mathcal{S}} &= \{\text{Mercury, Venus}\} \\
|Q|_{\mathcal{S}} &= \{\langle e, d \rangle : e \text{ is larger than } d\} \\
|R|_{\mathcal{S}} &= \{\langle e, d \rangle : e \text{ is farther from the sun than } d\}
\end{aligned}
$$

Let α be a variable assignment assigning Venus to x and Jupiter to y.

(1) Which of the following formulas are satisfied by α in $\mathcal{S}$? Explain your answers. 3

(i) $Qxy \wedge Ryx$

(ii) $Rxy \rightarrow Qyx$ 3

(iii) $\exists y\,(Py \leftrightarrow Ryx) \rightarrow Py$ 3

(2) Which of the following sentences are true in $\mathcal{S}$? Justify your answers as fully as possible.

(i) $\forall x\,\forall y\,(Rxy \rightarrow Qyx)$ 3

(ii) $\exists y\,\exists x\,(\neg(Py \wedge Qxy) \wedge \exists z(Pz \wedge Qyz))$ 4

(e) Disprove the following claims by providing counterexamples (no need to prove that your structure is a counterexample; you only need to specify the structure).

(i) $\exists x\,\exists y\,Qxy \vDash \neg\forall x\,\forall y\,Qxy$ 2

(ii) $\forall x\,(\neg Px \rightarrow \exists y\,(Rxy \vee Ryx)) \vDash \forall x\,\forall y\,(\neg Rxy \wedge \neg Ryx \rightarrow Px)$ 2

7. (a) For each of the sentences below explain the way in which it is ambiguous. If possible, reveal the ambiguity by formalising the sentence in two (or more) different ways.

   (i) Some letters can be found in every line. 2

   (ii) The first line in the book contains 15 letters. 2

   (iii) In a library in Oxford Albert saw the same book he had seen ten years earlier in a book shop in Budapest. Some edition of this book was known to all of Albert's friends, who haven't been there. 3

   (b) Formalise the following sentences as detailed as possible in the language $\mathcal{L}_2$ of predicate logic specifying your dictionary. Comment on any difficulties and points or interest.

   (i) Tim and Tom painted the wall. 2

   (ii) Tim ran and Tom ran quickly. 2

   (iii) Tom opened the box and Tim opened the tin with his pocket knife. 2

   (iv) Tim is looking for a pen. 2

   (v) Tom finds a pen. 2

   (c) Are the following arguments logically valid? Are they valid in predicate logic (without identity)? Explain your answer.

   (i) Tim is a logician. Tom is a logician. Therefore there is something they have in common. 2

   (ii) Tim is in Birmingham. Hence it's possible that Tim is in Birmingham. 2

   (iii) Tim isn't in London. Therefore Tom doesn't know that Tim is in London. 2

   (iv) Many students answered question 7 because they thought it was easier than the other questions. Therefore some students answered question 7. 2