In the previous example I showed:

$$P \leftrightarrow Q \models (P \wedge Q) \vee (\neg P \wedge \neg Q)$$

Now I want to find out whether

$$P \leftrightarrow Q \models (P \wedge Q) \vee (\neg P \wedge Q)$$

I write down the truth table as usual. The first steps are as in the previous example. Only the second line differs from the previous example.

I assume that the premiss is true and that the conclusion is false

P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
		T	F

If a disjunction $\phi \vee \psi$ is false then ϕ and ψ must be false.

Now there is no unique way to continue. So I distinguish two cases:

P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
		T	F_1 F F_1

$P \leftrightarrow Q$ can be true because P and Q are both true or because they are both false.

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
1			T_2 T T_2	F_1 F F_1
2			F_2 T F_2	F_1 F F_1

So P is true in the first line.

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
1			T_2 T T_2	T_3 F_1 F F_1
2			F_2 T F_2	F_1 F F_1

Since $P \wedge Q$ is false, Q must be false; but that contradicts my assumption that Q is true. So I put a question mark here: the line cannot be completed.

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
1			T_2 T T_2	T_3 F_1 ? F F_1
2			F_2 T F_2	F_1 F F_1

I turn to the second case: P and Q are both false…

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
1			T_2 T T_2	T_3 F_1 ? F F_1
2			F_2 T F_2	F_1 F F_3 F_1

and so $\neg P$ must be true.

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
1			T_2 T T_2	T_3 F_1 ? F F_1
2			F_2 T F_2	F_1 F T_4 F_3 F_1

As $\neg P \wedge Q$ is false and $\neg P$ is true, Q must be false, which is exactly what I have assumed.

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
1			T_2 T T_2	T_3 F_1 ? F F_1
2			F_2 T F_2	F_1 F T_4 F_3 F_1 F_5

I add the truth values for P and Q in $P \wedge Q$

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
1			T_2 T T_2	T_3 F_1 ? F F_1
2			F_2 T F_2	F_6 F_1 F T_4 F_3 F_1 F_5

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
1			T_2 T T_2	T_3 F_1 ? F F_1
2			F_2 T F_2	F_6 F_1 F_7 F T_4 F_3 F_1 F_5

So P and Q must be false.

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
1			T_2 T T_2	T_3 F_1 ? F F_1
2	F	F	F_2 T F_2	F_6 F_1 F_7 F T_4 F_3 F_1 F_5

I didn't find any problems with this line. To make sure that I haven't missed a clash of truth values, I re-calculate this line to make sure that the premiss is true and the conclusion is false if P and Q are both false.

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
1			T_2 T T_2	T_3 F_1 ? F F_1
2	F	F	F_2 T F_2	F_6 F_1 F_7 F T_4 F_3 F_1 F_5

Going through the line once more is important as otherwise you can't be sure you haven't overlooked a problem.

So I start from the truth value F for P and Q.

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
1			T_2 T T_2	T_3 F_1 ? F F_1
2	F	F	F_2 T F_2	F_6 F_1 F_7 F T_4 F_3 F_1 F_5
	F	F		

I copy the truth value F under every occurrence of P and Q.

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
1			T_2 T T_2	T_3 F_1 ? F F_1
2	F	F	F_2 T F_2	F_6 F_1 F_7 F T_4 F_3 F_1 F_5
	F	F	F F	F F F F

Then I complete the line in the usual way.

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
1			T_2 T T_2	T_3 F_1 ? F F_1
2	F	F	F_2 T F_2	F_6 F_1 F_7 F T_4 F_3 F_1 F_5
	F	F	F T F	F F F F

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
1			T_2 T T_2	T_3 F_1 ? F F_1
2	F	F	F_2 T F_2	F_6 F_1 F_7 F T_4 F_3 F_1 F_5
	F	F	F T F	F F F F F

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
1			T_2 T T_2	T_3 F_1 ? F F_1
2	F	F	F_2 T F_2	F_6 F_1 F_7 F T_4 F_3 F_1 F_5
	F	F	F T F	F F F T F F

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
1			T_2 T T_2	T_3 F_1 ? F F_1
2	F	F	F_2 T F_2	F_6 F_1 F_7 F T_4 F_3 F_1 F_5
	F	F	F T F	F F F T F F F

Yes, I have found a line in which the premiss is true and the conclusion is false. So I have proved that $P \leftrightarrow Q \not\models (P \wedge Q) \vee (\neg P \wedge Q)$.

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge Q)$
1			T_2 T T_2	T_3 F_1 ? F F_1
2	F	F	F_2 T F_2	F_6 F_1 F_7 F T_4 F_3 F_1 F_5
	F	F	F T F	F F F F T F F F