I want to prove the following by means of a partial truth table:

$$P \leftrightarrow Q \vDash (P \wedge Q) \vee (\neg P \wedge \neg Q)$$

I assume that the premiss is true and that the conclusion is false

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge \neg Q)$
			T	F

If a disjunction $\phi \vee \psi$ is false then ϕ and ψ must be false.

Now there is no unique way to continue. So I distinguish two cases:

	P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge \neg Q)$
			T	F_1 F F_1

$P \leftrightarrow Q \vDash (P \wedge Q) \vee (\neg P \wedge \neg Q)$

$P \leftrightarrow Q$ can be true because P and Q are both true or because they are both false.

	P	Q	P	$\leftrightarrow$	Q	$(P$	$\wedge$	$Q)$	$\vee$	$(\neg P$	$\wedge$	$\neg Q)$
1			T_2	T	T_2		F_1		F		F_1	
2			F_2	T	F_2		F_1		F		F_1	

So P is true in the first line.

	P	Q	P	$\leftrightarrow$	Q	$(P$	$\wedge$	$Q)$	$\vee$	$(\neg$	P	$\wedge$	$\neg$	$Q)$
1			T_2	T	T_2	T_3	F_1		F			F_1		
2			F_2	T	F_2		F_1		F			F_1		

Since $P \wedge Q$ is false, Q must be false; but that contradicts my assumption that Q is true. So I put a question mark here: the line cannot be completed.

	P	Q	P	$\leftrightarrow$	Q	$(P$	$\wedge$	$Q)$	$\vee$	$(\neg$	P	$\wedge$	$\neg$	$Q)$
1			T_2	T	T_2	T_3	F_1	?	F			F_1		
2			F_2	T	F_2		F_1		F			F_1		

I turn to the second case: *P* and *Q* are both false…

	P	Q	P	$\leftrightarrow$	Q	$(P$	$\wedge$	$Q)$	$\vee$	$(\neg$	P	$\wedge$	$\neg$	$Q)$
1			T_2	T	T_2	T_3	F_1	?	F			F_1		
2			F_2	T	F_2		F_1		F		F_3	F_1		

and so $\neg P$ must be true.

	P	Q	P	$\leftrightarrow$	Q	$($	P	$\wedge$	Q	$)$	$\vee$	$($	$\neg$	P	$\wedge$	$\neg$	Q	$)$
1			T_2	T	T_2		T_3	F_1	?		F				F_1			
2			F_2	T	F_2			F_1			F		T_4	F_3	F_1			

As $\neg P \wedge \neg Q$ is false and $\neg P$ is true, $\neg Q$ must be false.

	P	Q	P	$\leftrightarrow$	Q	$(P$	$\wedge$	$Q)$	$\vee$	$(\neg$	P	$\wedge$	$\neg$	$Q)$
1			T_2	T	T_2	T_3	F_1	?	F			F_1		
2			F_2	T	F_2		F_1		F	T_4	F_3	F_1	F_5	

Since $\neg Q$ is false Q ought to be true, but that contradicts our assumption that Q is false; so the line cannot be completed.

	P	Q	P	$\leftrightarrow$	Q	$($	P	$\land$	Q	$)$	$\lor$	$($	$\neg$	P	$\land$	$\neg$	Q	$)$
1			T_2	T	T_2		T_3	F_1	?		F				F_1			
2			F_2	T	F_2			F_1			F		T_4	F_3	F_1	F_5	?	

Thus, there is no line in the truth table where the premiss is true and the conclusion is false; so I have proved $P \leftrightarrow Q \vDash (P \wedge Q) \vee (\neg P \wedge \neg Q)$

	P	Q	P	$\leftrightarrow$	Q	$(P$	$\wedge$	$Q)$	$\vee$	$(\neg$	P	$\wedge$	$\neg$	$Q)$
1			T_2	T	T_2	T_3	F_1	?	F			F_1		
2			F_2	T	F_2		F_1		F	T_4	F_3	F_1	F_5	?