Using a partial truth table I will find out whether

$$(P \to (Q \to R)) \to (P \wedge Q \to R)$$

is a tautology. A sentence of the language of propositional logic is a tautology (logically true) if and only if the main column has T in every line of the truth value (that is, if and only if the sentence is true in any $\mathcal{L}_1$-structure).

As it stands, the sentence

$$(P \to (Q \to R)) \to (P \wedge Q \to R)$$

is merely in abbreviated form. Before drawing a truth table one should know how the sentence has been built up. The sentence with all its brackets in place reads as follows:

$$((P \to (Q \to R)) \to ((P \wedge Q) \to R))$$

One doesn't have to add the brackets. But they may help to understand the structure of the sentence when one is calculating truth values in the table. In the truth table I have dropped the outer brackets.

I assume that there is a line in which the sentence comes out false. So I write an F in the main column.

P	Q	R	$(P \to (Q \to R)) \to ((P \wedge Q) \to R)$
			F

The truth table for $\to$ is:

ϕ	ψ	$(\phi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

So $\phi \to \psi$ is false only if ϕ is true and ψ is false. So the sentence in front of $\to$ must be true…

P	Q	R	$(P \to (Q \to R)) \to ((P \wedge Q) \to R)$
			T_1 F

and the sentence following the arrow must be false.

P	Q	R	$(P \to (Q \to R)) \to ((P \wedge Q) \to R)$
			T_1 F F_2

The sentence with T under it could be true for three reasons: there are three lines in the table of $\to$ that have a T. If I continued here I would have to distinguish three cases (This wouldn't lead to an incorrect result; it would just make the calculations more awkward). I better turn to the other sentence. It's again a sentence with $\to$. It's false and it can be false only if the sentence in front of $\to$ is true…

P	Q	R	$(P \to (Q \to R)) \to ((P \wedge Q) \to R)$
			T_1 F T_3 F_2

and the following sentence is false.

P	Q	R	$(P \to (Q \to R)) \to ((P \wedge Q) \to R)$
			T_1 $\quad$ F $\quad$ T_3 $\quad$ F_2 F_4

Now I have a T for $P \wedge Q$. The truth table for $\wedge$ is

ϕ	ψ	$(\phi \wedge \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

So for a sentence $\phi \wedge \psi$ there is only one way to be true: ϕ and ψ must both be true. Therefore I write T under P and…

P	Q	R	$(P \to (Q \to R)) \to ((P \wedge Q) \to R)$
			T_1 F T_5 T_3 F_2 F_4

T under Q

P	Q	R	$(P \to (Q \to R)) \to ((P \wedge Q) \to R)$
			T_1 $\quad$ F $\quad$ T_5 T_3 T_6 F_2 F_4

So P gets truth value T,

P	Q	R	$(P \to (Q \to R)) \to ((P \wedge Q) \to R)$
			T$_7$ T$_1$ F T$_5$ T$_3$ T$_6$ F$_2$ F$_4$

Q receives T

P	Q	R	$(P \to (Q \to R)) \to ((P \wedge Q) \to R)$
			T_7 T_1 T_8 F T_5 T_3 T_6 F_2 F_4

and R gets F.

P	Q	R	$(P \to (Q \to R)) \to ((P \wedge Q) \to R)$
			T_7 T_1 T_8 F_9 F T_5 T_3 T_6 F_2 F_4

So $Q \to R$ must be false; but that cannot be because $P \to (Q \to R)$ and P are true, and, thus, $Q \to R$ must be true. So I can't complete this line in the truth table. Therefore there cannot be a line in the full truth table with an F in the main column. So there are only T in the main column and the sentence $(P \to (Q \to R)) \to (P \wedge Q \to R)$ is a tautology.

P	Q	R	$(P \to (Q \to R)) \to ((P \wedge Q) \to R)$
			T_7 T_1 T_8 ? F_9 F T_5 T_3 T_6 F_2 F_4