

I want to show that

$$\forall x\forall y\forall z(Rxy \rightarrow (Ryz \rightarrow Pxz)), Rab, Rbc \vdash \exists x\exists y(Rxy \wedge \exists zPxz).$$

$$\forall x \forall y \forall z (Rxy \rightarrow (Ryz \rightarrow Pxz))$$

I write down the premiss  
 $\forall x \forall y \forall z (Rxy \rightarrow (Ryz \rightarrow Pxz))$ .

$$\frac{\forall x \forall y \forall z (Rxy \rightarrow (Ryz \rightarrow Pxz))}{\forall y \forall z (Ray \rightarrow (Ryz \rightarrow Paz))}$$

and apply  $\forall$ Elim once ...

$$\frac{\frac{\forall x \forall y \forall z (Rxy \rightarrow (Ryz \rightarrow Pxz))}{\forall y \forall z (Ray \rightarrow (Ryz \rightarrow Paz))}}{\forall z (Rab \rightarrow (Rbz \rightarrow Paz))}$$

... and twice.

$$\frac{\frac{\frac{\forall x \forall y \forall z (Rxy \rightarrow (Ryz \rightarrow Pxz))}{\forall y \forall z (Ray \rightarrow (Ryz \rightarrow Paz))}}{\forall z (Rab \rightarrow (Rbz \rightarrow Paz))}}{Rab \rightarrow (Rbc \rightarrow Pac)}$$

... and once more;

$$\frac{\frac{\frac{\forall x \forall y \forall z (Rxy \rightarrow (Ryz \rightarrow Pxz))}{\forall y \forall z (Ray \rightarrow (Ryz \rightarrow Paz))}}{\forall z (Rab \rightarrow (Rbz \rightarrow Paz))}}{\frac{Rab}{Rab \rightarrow (Rbc \rightarrow Pac)}}}{Rbc \rightarrow Pac}$$

so I can use the other premisses  
by applying  $\rightarrow$ Elim,

$$\frac{\frac{\frac{\frac{\forall x \forall y \forall z (Rxy \rightarrow (Ryz \rightarrow Pxz))}{\forall y \forall z (Ray \rightarrow (Ryz \rightarrow Paz))}}{\forall z (Rab \rightarrow (Rbz \rightarrow Paz))}}{\frac{Rab}{Rab \rightarrow (Rbc \rightarrow Pac)}}}{\frac{Rbc}{Rbc \rightarrow Pac}} \quad Pac$$

and similarly for  $Rbc$ .

$$\frac{\frac{\frac{\frac{Rab}{Rab \rightarrow (Rbc \rightarrow Pac)}}{Rbc \rightarrow Pac}}{Pac}}{\exists z Paz}}$$

I apply  $\exists$ Intro ...



$$\frac{\frac{\frac{\frac{Rab}{Rbc} \quad Rab}{Rbc \rightarrow Pac} \quad Rab \rightarrow (Rbc \rightarrow Pac)}{Rbc \rightarrow Pac} \quad \frac{\forall x \forall y \forall z (Rxy \rightarrow (Ryz \rightarrow Pxz))}{\forall y \forall z (Ray \rightarrow (Ryz \rightarrow Paz))}}{\forall z (Rab \rightarrow (Rbz \rightarrow Paz))}}{Rab \wedge \exists z Paz}$$

and combine the resulting sentence with the premiss *Rab*..

$$\frac{\frac{\frac{\frac{Rab}{Rbc}}{Rab} \quad \frac{Rab}{Rab \rightarrow (Rbc \rightarrow Pac)}}{Rbc \rightarrow Pac}}{Pac}}{\exists z Paz}}{Rab \wedge \exists z Paz}}{\exists y (Ray \wedge \exists z Paz)}$$

Finally two applications of  $\exists$ Intro yield the conclusion.

$$\frac{\frac{\frac{\frac{\frac{Rab}{Rbc}}{Rab}}{Rab \rightarrow (Rbc \rightarrow Pac)}}{Rbc \rightarrow Pac}}{Pac}}{\exists z Paz}}{\frac{Rab \wedge \exists z Paz}{\exists y (Ray \wedge \exists z Paz)}}}{\exists x \exists y (Rxy \wedge \exists z Pxz)}$$