

We want to show that $\exists y \forall x Rxy \vdash \forall x \exists y Rxy$.

$\exists y \forall x Rxy$ $\forall x Rxb$

I write down the premiss $\exists y \forall x Rxy$ as an assumption. Now in the hope of being able to apply \exists Elim I assume $\forall x Rxb$. I could have used any other constant instead of b .

$\exists y \forall x Rxy$

$$\frac{\forall x Rxb}{Rab}$$

I apply \forall Elim. I don't use the constant b for this, because later I intend to apply \forall Intro and thus the constant shouldn't occur in any undischarged assumption.

$\exists y \forall x Rxy$

$$\frac{\forall x Rxb}{\frac{Rab}{\exists y Ray}}$$

Now I can reintroduce the existential quantifier by applying \exists Intro.

$$\exists y \forall x Rxy \quad \frac{\frac{\forall x Rxb}{Rab}}{\exists y Ray}}{\forall x \exists y Rxy}$$

Since a doesn't occur in the (undischarged) assumption $\forall x Rxb$ I can apply \forall Intro. Now I have arrived at the conclusion...

$$\frac{\exists y \forall x Rxy \quad \frac{\frac{[\forall x Rxb]}{Rab}}{\exists y Ray}}{\forall x \exists y Rxy}}{\forall x \exists y Rxy}$$

I just to apply \exists Elim to discharge the assumption $\forall x Rxb$. Only $\exists y \forall x Rxy$ remains as undischarged, but this is no problem, because $\exists y \forall x Rxy$ is a premiss.

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