

We want to show that  $\exists y \forall x Rxy \vdash \forall x \exists y Rxy$ .

$\exists y \forall x Rxy$  $\forall x Rxb$ 

I write down the premiss  $\exists y \forall x Rxy$  as an assumption. Now in the hope of being able to apply  $\exists$ Elim I assume  $\forall x Rxb$ . I could have used any other constant instead of  $b$ .

$\exists y \forall x Rxy$ 

$$\frac{\forall x Rxb}{Rab}$$

I apply  $\forall$ Elim. I don't use the constant  $b$  for this, because later I intend to apply  $\forall$ Intro and thus the constant shouldn't occur in any undischarged assumption.

$\exists y \forall x Rxy$ 

$$\frac{\forall x Rxb}{\frac{Rab}{\exists y Ray}}$$

Now I can reintroduce the existential quantifier by applying  $\exists$ Intro.

$$\begin{array}{c} \exists y \forall x Rxy \\ \frac{\frac{\forall x Rxb}{\frac{Rab}{\exists y Ray}}}{\forall x \exists y Rxy} \end{array}$$

Since  $a$  doesn't occur in the (undischarged) assumption  $\forall x Rxb$  I can apply  $\forall$ Intro. Now I have arrived at the conclusion...

$$\frac{\exists y \forall x Rxy \quad \frac{\frac{[\forall x Rxb]}{Rab}}{\exists y Ray}}{\forall x \exists y Rxy}}{\forall x \exists y Rxy}$$

I just to apply  $\exists$ Elim to discharge the assumption  $\forall x Rxb$ . Only  $\exists y \forall x Rxy$  remains as undischarged, but this is no problem, because  $\exists y \forall x Rxy$  is a premiss.

Thanks to Jeremy Stone for spotting a mistake in an earlier version of this file.