

I try to prove that $\neg P \vee Q, P \vee \neg Q \vdash P \leftrightarrow Q$.

The first premiss is $\neg P \vee Q$. I try to apply \vee Elim.

$$\neg P \vee Q$$

So I assume $\neg P$ and Q .

$$\neg P \vee Q \quad \neg P$$

$$Q$$

I want to prove Q from P . Combining this with a proof of P from Q will allow me to prove the conclusion. To derive Q from P I assume P .

	$\neg P$	P	
$\neg P \vee Q$			Q

I apply \neg -Elim to $\neg Q$, which has not been assumed. This gives Q .

$$\neg P \vee Q \quad \frac{\neg P \quad P}{Q} \quad Q$$

Now I can apply \vee Elim and discharge the assumptions $\neg P$ and Q .

$$\frac{\neg P \vee Q \quad \frac{[\neg P] \quad P}{Q} \quad [Q]}{Q}$$

To prove P from Q I proceed in an analogous way.

$$\frac{\neg P \vee Q \quad \frac{[\neg P] \quad P}{Q}}{Q} \quad [Q]$$

$$\frac{P \vee \neg Q \quad \frac{[\neg Q] \quad Q}{P}}{P} \quad [P]$$

Now I can apply \leftrightarrow Intro and discharge P in the proof on the left and Q in the proof on the right.

$$\frac{\frac{\neg P \vee Q}{\frac{\frac{[\neg P] \quad [P]}{Q}}{Q}}{Q} \quad [Q]}{P \leftrightarrow Q} \quad \frac{\frac{P \vee \neg Q}{\frac{[\neg Q] \quad [Q]}{P}}{P}}{P}}{P \leftrightarrow Q}}$$