We want to show that $P \vee \neg Q, R \to \neg P \vdash Q \to \neg R$.

At first I explain how to find the proof.

The first premiss is $P \lor \neg Q$. I try to apply $\lor$Elim. Thus I assume P and $\neg Q$

P

$\neg Q$

$P \lor \neg Q$

I need to show $Q \to \neg R$. I hope to get this by an application of →Intro. So I should try to prove $\neg R$ under the assumption Q. First I look at the case P. I hope to get $\neg R$ by ¬Intro; so I assume R.

R

P $\qquad\qquad\qquad\qquad$ $\neg Q$

$P \vee \neg Q$

R is can be combined with the premiss $R \rightarrow \neg P$ to obtain $\neg P$.

$$\dfrac{R \qquad R \rightarrow \neg P}{\neg P}$$

P $\qquad$ $\neg Q$

$P \vee \neg Q$

$\neg P$ yields a contradiction with the assumption P. So I can conclude $\neg R$ and discharge R by applying $\neg$Intro.

$$P \vee \neg Q \qquad \dfrac{P \qquad \dfrac{[R] \qquad R \to \neg P}{\neg P}}{\neg R} \qquad \neg Q$$

I turn to the second case with $\neg Q$ as assumption. I want to arrive at $\neg R$ under the assumption Q.

$$P \vee \neg Q \qquad \dfrac{P \qquad \dfrac{[R] \qquad R \to \neg P}{\neg P}}{\neg R} \qquad Q \qquad \neg Q$$

$\neg R$ can be obtained from Q and $\neg Q$ by an application of ¬Intro, although R hasn't been assumed.

$$P \vee \neg Q \qquad \dfrac{P \qquad \dfrac{[R] \qquad R \to \neg P}{\neg P}}{\neg R} \qquad \dfrac{Q \qquad \neg Q}{\neg R}$$

Now I can apply $\vee$Elim to discharge the assumptions P and $\neg Q$.

$$\dfrac{P \vee \neg Q \qquad \dfrac{[P] \qquad \dfrac{[R] \qquad R \to \neg P}{\neg P}}{\neg R} \qquad \dfrac{Q \qquad [\neg Q]}{\neg R}}{\neg R}$$

I discharge the assumption Q by →Intro. This completes the proof.

$$\dfrac{\dfrac{P \lor \neg Q \qquad \dfrac{[P] \qquad \dfrac{[R] \qquad R \to \neg P}{\neg P}}{\neg R} \qquad \dfrac{[Q] \qquad [\neg Q]}{\neg R}}{\neg R}}{Q \to \neg R}$$

Alternatively I could have applied $\to$Intro first and then $\lor$Elim. The proof looks then as follows…

$$\dfrac{\dfrac{P \lor \neg Q \qquad \dfrac{[P] \qquad \dfrac{[R] \qquad R \to \neg P}{\neg P}}{\neg R} \qquad \dfrac{[Q] \qquad [\neg Q]}{\neg R}}{\neg R}}{Q \to \neg R}$$

$$\dfrac{P \lor \neg Q \qquad \dfrac{\dfrac{[P] \qquad \dfrac{[R] \qquad R \to \neg P}{\neg P}}{\neg R}}{Q \to \neg R} \qquad \dfrac{\dfrac{[Q] \qquad [\neg Q]}{\neg R}}{Q \to \neg R}}{Q \to \neg R}$$