

I will give a proof for the following claim:

$$\exists x \neg x = x \vdash \forall x Px$$

I write down the premiss as an assumption.

$$\exists x \neg x = x$$

In order to apply  $\exists$ Elim later I assume  $\neg a = a$ .

$$\exists x \neg x = x$$

$$\neg a = a$$

I assume  $a = a$  and immediately discharge this assumption in accordance with =Intro.

$$\exists x \neg x = x \quad \neg a = a \quad [a = a]$$

So I have a contradiction. Applying  $\neg$ -Intro to  $\neg \forall x Px$  (which has never been assumed) I infer  $\forall x Px$ .

$$\exists x \neg x = x \quad \frac{\neg a = a \quad [a = a]}{\forall x Px}$$

I apply  $\exists$ Elim and discharge the assumption  $\neg a = a$ .

$$\frac{\exists x \neg x = x \quad \frac{[\neg a = a] \quad [a = a]}{\forall x Px}}{\forall x Px}$$

$$\frac{\exists x \neg x = x \quad \frac{[\neg a = a] \quad [a = a]}{\forall x Px}}{\forall x Px}$$