INTRODUCTION TO LOGIC

Lecture 7

Formalisation in Predicate Logic

Dr. James Studd

'Contrariwise,' continued Tweedledee,
'if it was so, it might be;
and if it were so, it would be;
but as it isn't, it ain't.
That's logic'
Lewis Carroll
Through the Looking-Glass

Outline

(1) Review of adequacy.

(2) Logical properties of English sentences.

(3) Further issues in predicate formalisation.

Recap: Adequacy

Two notions of consequence coincide.

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Let Γ be a set of $\mathcal{L}_2$-sentences and ϕ a $\mathcal{L}_2$-sentence

Definition: provable (syntactic)

$\Gamma \vdash \phi$ iff there is a proof of ϕ with only sentences in Γ as non-discharged assumptions.

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Definition: valid (semantic)

$\Gamma \models \phi$ iff there is no $\mathcal{L}_2$-structure in which all sentences in Γ are true and ϕ is false.

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Adequacy theorem (Soundness and Completeness)

$\Gamma \vdash \phi$ iff $\Gamma \models \phi$

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Step (i) Formalise the argument in $\mathcal{L}_2$.

Step (ii) Prove the formalised argument in Natural Deduction.

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Method to establish non-validity

Step (i) Formalise the argument in $\mathcal{L}_2$.

Step (ii) Construct a counterexample. (An $\mathcal{L}_2$-structure in which the premisses are true and the conclusion is false.)

Unfinished business

Show that argument 2 is valid

(1) Zeno is a tortoise.
(2) All tortoises are toothless.
Therefore, (C) Zeno is toothless.

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Step (i): formalise

(1) Ta
(2) $\forall x(Tx \to Lx)$
(C) La

Dictionary: a: Zeno. T:. . . is a tortoise. L: . . . is toothless

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Need to show: $Ta, \forall x(Tx \to Lx) \models La$.

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That completes the proof
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The English argument about Zeno is valid in predicate logic.

Exercise: show that the following argument is valid

All concrete objects are located in space. The number 5 isn't located in space. So the number 5 isn't a concrete object.

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Premiss 1: $\forall x\,(Cx \to Lx)$

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Premiss 1: $\forall x\,(Cx \to Lx)$
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So the argument is valid in predicate logic.

Note on partial formalisation

To establish validity: partial formalisation may suffice.

	Less detailed (I)
Premiss 1	$\forall x\,(Cx \to Lx)$
Premiss 2	$\neg La$
Conclusion	$\neg Ca$

Note on partial formalisation

To establish validity: partial formalisation may suffice.

	Less detailed (I)	More detailed (II)
Premiss 1	$\forall x\,(Cx \to Lx)$	$\forall x\,(Cx \to L^2xb)$
Premiss 2	$\neg La$	$\neg L^2ab$
Conclusion	$\neg Ca$	$\neg Ca$

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Both (I) and (II) are fine.

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To establish validity: partial formalisation may suffice.

	Less detailed (I)	More detailed (II)	Not detailed enough (III)
Premiss 1	$\forall x\,(Cx \to Lx)$	$\forall x\,(Cx \to L^2xb)$	A
Premiss 2	$\neg La$	$\neg L^2ab$	S
Conclusion	$\neg Ca$	$\neg Ca$	C

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Both (I) and (II) are fine. (III) is not.

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Both (I) and (II) are fine. (III) is not.

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NB: to show non-validity: full formalisation required.

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Conclusion: Ka.

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Step (ii): construct a counterexample.

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The premisses are true, and the conclusion is false in $\mathcal{A}$.
So $\mathcal{A}$ is a counterexample.

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Step (i) formalise the sentence as a sentence ϕ of $\mathcal{L}_2$.
Step (ii) prove that $\vdash \phi$.

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Step (i) formalise the sentence as a sentence ϕ of $\mathcal{L}_2$.
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Example: show the sentence is a contradiction.

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Step (i): formalise

Paraphrase: Some x is such that (x is bigger than everything and x is not bigger than itself)

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Paraphrase: Some x is such that (x is bigger than everything and x is not bigger than itself)

- x is bigger than everything: $\forall y Bxy$

Dictionary: B: . . . is bigger than . . . .

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Step (i): formalise

Paraphrase: Some x is such that (x is bigger than everything and x is not bigger than itself)

- x is bigger than everything: $\forall y Bxy$
- x is not bigger than itself: $\neg Bxx$

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We need to show: $\vdash \neg\exists x(\forall y Bxy \wedge \neg Bxx)$

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Further issues: scope ambiguity

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Paraphrases

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(1) $\forall x(Px \to \exists y(My \land Kxy))$

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Issue 2: variable arity.

Example: formalise in $\mathcal{L}_2$ as a valid argument.

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Formalise in $\mathcal{L}_2$ as a valid argument.

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Manny is eating the scrambled egg out of something with something and Manny is eating something out of his shoe with something.

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Issue 3: adverbs

Example: formalise in $\mathcal{L}_2$ as a valid argument.

Usain ran quickly; so Usain ran. 45

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Example: formalise in $\mathcal{L}_2$ as a valid argument.

Usain ran quickly; so Usain ran. 45

The following formalisation is clearly not valid.

Premiss: Qb. Conclusion: Rb.

Dictionary: b: Usain. Q: ...ran quickly. R: ...ran.

Issue 3: adverbs

Example: formalise in $\mathcal{L}_2$ as a valid argument.

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The following formalisation is clearly not valid.

Premiss: Qb. Conclusion: Rb.

Dictionary: b: Usain. Q: …ran quickly. R: …ran.

But there is a (somewhat contrived) way to formalise it.

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Issue 3: adverbs

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The following is valid:

Premiss: $\exists x(R_1x \land Pbx \land Q_1x)$. Conclusion: $\exists x(R_1x \land Pbx)$.

Issue 4: non-extensionality

Example

Miles wants to live in Oxford. Oxford is a city with high levels of air pollution. Therefore Miles wants to live in a city with high levels of air pollution.

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Yet the obvious $\mathcal{L}_2$-formalisation is valid.

Premiss 1: Lmo.
Premiss 2: Po.
Conclusion: $\exists x\,(Lmx \wedge Px)$.

L:	… wants to live in …
P:	… is a city with high levels of air pollution
m:	Miles
o:	Oxford

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What has gone wrong?

Extensionality of $\mathcal{L}_2$

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$\mathcal{L}_2$-structures assign extensions to expressions.

$\mathcal{L}_2$-*expression*	*extension*
constant	object
sentence	truth-value
unary predicate	set
binary predicate	set of pairs

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They have the following feature.

Extensionality

In a $\mathcal{L}_2$-structure, the extension of a sentence depends only on the extensions of its constituent expressions.

Let $\mathcal{A}$ be an $\mathcal{L}_2$-structure, ϕ an $\mathcal{L}_2$-sentence.

Extensionality in $\mathcal{L}_2$

Replacing an expression in ϕ for another with the same extension in $\mathcal{A}$ leaves the extension (truth-value) of ϕ in $\mathcal{A}$ unchanged.

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- ... knows that ... wears a cape
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'... knows that Superman wears a cape' *is* extensional.

http://logicmanual.philosophy.ox.ac.uk