INTRODUCTION TO LOGIC

Lecture 5

The Semantics of Predicate Logic

Dr. James Studd

We could forget about philosophy.
Settle down and maybe get into semantics.
Woody Allen
'Mr. Big'

Outline

1. Validity.
2. Semantics for simple English sentences.
3. Semantics for $\mathcal{L}_2$-formulae.
4. $\mathcal{L}_2$-structures.

What of argument 2?

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(2) All tortoises are toothless.
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What is it for this $\mathcal{L}_2$-argument to be valid?

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It remains to define: $\mathcal{L}_2$-structure, truth in an $\mathcal{L}_2$-structure

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... because Russell has the property of *being a philosopher*.
... because |'Bertrand Russell'| has |'is a philosopher'|.

Notation

When e is an expression, we write $|e|$ for its semantic value

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In other words:

|'Alonzo Church reveres Bertrand Russell'| = T iff
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We'll take this one step further, by saying more about properties and relations.

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Example

The property of *being a philosopher*

$$= \text{the set of philosophers}$$
$$= \{d : d \text{ is a philosopher}\}$$
$$= \{\text{Descartes, Kant, Russell, } \ldots\}$$

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Notation: $|e|_{\mathcal{A}}$ is the semantic value of e in $\mathcal{L}_2$-structure $\mathcal{A}$.

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What object each variable denotes is specified with a variable assignment.

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A variable assignment assigns an object to each variable.

One can think of a variable assignment as an infinite list

Example: the assignment α.

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Mercury	Venus	Venus	Neptune	Mars	Venus	Mars	

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Similarly for other atomic formulae.

Worked example

Let $\mathcal{L}_2$-structure $\mathcal{A}$ be such that:

- $|a|_{\mathcal{A}} =$ Alonzo Church
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- $|P|_{\mathcal{A}} = \{$Frege, Russell$\}$
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This is correct but the general case is more complex.

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To progress any further we need to be able evaluate $\exists y Rxy$ under an assignment α of an object to x.

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$\mathcal{L}_2$-expression	*semantic value in $\mathcal{A}$*
constant: a	object: $\lvert a\rvert_\mathcal{A}$
sentence letter: P	truth-value: $\lvert P\rvert_\mathcal{A}$ (= T or F)
unary predicate: P^1	unary relation: $\lvert P^1\rvert_\mathcal{A}$ (i.e. a set)
binary predicate: P^2	binary relation: $\lvert P^2\rvert_\mathcal{A}$ (a set of pairs)
ternary predicate: P^3	ternary relation: $\lvert P^3\rvert_\mathcal{A}$ (a set of triples)
etc. etc.	

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etc.

The semantics for connectives are just like those for $\mathcal{L}_1$.

Semantics for connectives

(ii) $|\neg\phi|^\alpha_\mathcal{A} = \mathrm{T}$ if and only if $|\phi|^\alpha_\mathcal{A} = \mathrm{F}$.

(iii) $|\phi \wedge \psi|^\alpha_\mathcal{A} = \mathrm{T}$ if and only if $|\phi|^\alpha_\mathcal{A} = \mathrm{T}$ and $|\psi|^\alpha_\mathcal{A} = \mathrm{T}$.

(iv) $|\phi \vee \psi|^\alpha_\mathcal{A} = \mathrm{T}$ if and only if $|\phi|^\alpha_\mathcal{A} = \mathrm{T}$ or $|\psi|^\alpha_\mathcal{A} = \mathrm{T}$.

(v) $|\phi \rightarrow \psi|^\alpha_\mathcal{A} = \mathrm{T}$ if and only if $|\phi|^\alpha_\mathcal{A} = \mathrm{F}$ or $|\psi|^\alpha_\mathcal{A} = \mathrm{T}$.

(vi) $|\phi \leftrightarrow \psi|^\alpha_\mathcal{A} = \mathrm{T}$ if and only if $|\phi|^\alpha_\mathcal{A} = |\psi|^\alpha_\mathcal{A}$.

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(vii) $|\forall v\, \phi|^{\alpha}_{\mathcal{A}} = \mathrm{T}$ if and only if $|\phi|^{\beta}_{\mathcal{A}} = \mathrm{T}$ for all variable assignments β over $\mathcal{A}$ differing from α in v at most.

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(viii) $|\exists v\,\phi|^{\alpha}_{\mathcal{A}} = \mathrm{T}$ if and only if $|\phi|^{\beta}_{\mathcal{A}} = \mathrm{T}$ for at least one variable assignment β over $\mathcal{A}$ differing from α in v at most.

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equivalently: $|\phi|^{\alpha}_{\mathcal{A}} = \mathrm{T}$ for some variable assignment α over $\mathcal{A}$.

http://logicmanual.philosophy.ox.ac.uk