INTRODUCTION TO LOGIC

Lecture 4

The Syntax of Predicate Logic

Dr. James Studd

I counsel you, dear friend, in sum,
That first you take collegium logicum.
Your spirit's then well broken in for you,
In Spanish boots laced tightly to,
That you henceforth may more deliberately keep
The path of thought and straight along it creep,
And not perchance criss-cross may go,
A- will-o'-wisping to and fro.
Then you'll be taught full many a day
What at one stroke you've done alway,
Like eating and like drinking free,
It now must go like: One! Two! Three!

Goethe, Faust I

Outline

1. Introduction to $\mathcal{L}_2$
2. Straightforward predicate formalisation
3. The syntax of $\mathcal{L}_2$

Recall argument 2 from week 1.

Argument 2

(1) Zeno is a tortoise.
(2) All tortoises are toothless.
Therefore, (C) Zeno is toothless.

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Argument 2 **Valid**

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Propositional Formalisation

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Propositional Formalisation

(1) T

Dictionary: T: Zeno is a tortoise.

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(1) T

(2) A

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Propositional Formalisation

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Dictionary: T: Zeno is a tortoise.
A: All tortoises are toothless. L: Zeno is toothless.

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Dictionary: T: Zeno is a tortoise.
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But $T, A \not\models L$.

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Argument 2 **Valid**

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Therefore, (C) Zeno is toothless.

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Propositional Formalisation **Not Valid**

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(2) A

(C) L

Dictionary: T: Zeno is a tortoise.
A: All tortoises are toothless. L: Zeno is toothless.

But $T, A \not\models L$.

Subject-Predicate form.

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The language we use is the language of predicate logic: $\mathcal{L}_2$

Designators/Constants

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$\mathcal{L}_2$ constants

- a, b, c, etc.

Predicates

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- ______ is a tortoise

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English predicates

- is a tortoise
- London is bigger than the capital of France

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- is a tortoise
- is bigger than
- John opened the bottle with his keyring

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- is a tortoise — Arity: 1
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Some $\mathcal{L}_2$-sentences

Examples: P^1a

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Examples: P^1a, R^2ab, R^2aa, $P^4_{23}abca$

Predicate formalisation

We can now formalise the first premise

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Dictionary: a: Zeno. P^1: …is a tortoise.

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More examples **Formalisation**

(1) Alice paid Beatrice.

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(1) Alice paid Beatrice.	$P^2 ab$
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(4) Alice paid Beatrice or was paid by Beatrice	

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Argument 2 revisited, again

What about the second premiss?

All tortoises are toothless

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Quantifier / Pronoun / Predicate

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Everything is such that (Quantifier) it (Pronoun) has mass (Predicate)

Pronouns/Variables

In English, pronouns are expressions used instead of nouns

English pronouns

'she', 'him', 'it'

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Lazy use

Tom likes Miranda Fitzwilliam-Carter and she likes him.

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Tom likes Miranda Fitzwilliam-Carter and she likes him.

The pronouns each refer to a particular person

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The pronoun cannot be replaced by the noun to which it refers back (without changing the meaning).

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Lazy use

Tom likes Miranda Fitzwilliam-Carter and Miranda Fitzwilliam-Carter likes Tom.

The pronouns each refer to a particular person

Quantificational use

If a girl likes Tom, Tom likes her.

The pronoun cannot be replaced by the noun to which it refers back (without changing the meaning).

The pronoun 'her' does not refer to a single person: it is used to make a general claim.

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Quantifiers

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Formalising general sentences

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Dictionary: M: …has mass.

Note: it's fine to omit the arity index when formalising.

More examples

(1) Everything has mass.

(2) Something has mass.

(3) Some person has mass.

(4) Every person has mass.

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Common variants

$\mathcal{L}_2$	standard connective	some other formulations
$\forall x$	everything	every, all, any, a
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Examples

(1) A dog barked.

(2) A dog barks.

Dictionary. D: …is a dog. B_1: …barked. B_2: …barks

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Paraphrase (of the most natural readings)

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(1) A dog barked.	$\exists x(Dx \wedge B_1 x)$
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(1) Some dog barked
Some x is such that (x is a dog and x barked).

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Every x is such that (if x is dog, then x barks).

Quantifiers at the end of sentences

Examples

(1) Everyone loves Zuleika

(2) Zuleika loves everyone

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Paraphrases

(1) Every x is such that
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(2) Everyone is loved by Zuleika

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Paraphrases

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We use the same trick to deal with multiple quantifiers.

Example

Someone loves everyone

We use the same trick to deal with multiple quantifiers.

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Dictionary: P: ... is a person. L: ... loves ....

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Some x is such that (x is a person and x loves everyone)

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Some x is such that (x is a person and x loves everyone)

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Some x is such that

(x is a person and everything is such that:

(if it is a person, then it is loved by x))

We use the same trick to deal with multiple quantifiers.

Example

Someone loves everyone

Dictionary: P: ... is a person. L: ... loves ....

Paraphrases

Some x is such that (x is a person and x loves everyone)

Some x is such that (x is a person and everyone is loved by x)

Some x is such that
(x is a person and everything is such that:
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Example **Formalisation**

Someone loves everyone $\exists x(Px \land \forall y(Py \to Lxy))$

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The syntax of $\mathcal{L}_2$

Here's the official syntax of $\mathcal{L}_2$ 40

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Definition (Predicate letters)

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If Z is a predicate letter of arity n and each of $t_1, \ldots, t_n$ is a variable or a constant, then $Zt_1 \ldots t_n$ is an atomic formula of $\mathcal{L}_2$.

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3. If v is a variable and ϕ is a formula, then $\forall v\, \phi$ and $\exists v\, \phi$ are formulae of $\mathcal{L}_2$.

Worked example: which of these are $\mathcal{L}_2$-formulae?

(i) Pa

(ii) Q^3axy

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(iv) P

(v) $(P \to Q)$

(vi) $\forall x P^2 ax$

(vii) $\forall x \forall x P^1 x$

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An $\mathcal{L}_2$ formula ϕ is a sentence if every occurrence of each variable v in ϕ is ‘bound’ by a quantifier $\exists v$ or $\forall v$.

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- But you should stick to letters early in the alphabet for constants and letters late in the alphabet (start with: x, y, z) for variables.

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