

INTRODUCTION TO LOGIC

Lecture 3

Formalisation in Propositional Logic

Dr. James Studd

There is no other way to learn the truth
than through logic
[Averroes](#)

Outline

- ① Truth-functionality
- ② Formalisation
- ③ Complex sentences
- ④ Ambiguity
- ⑤ Validity of English arguments

English connectives

Recall that connectives join one or more sentences together to make compound sentences.

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These correspond to the connectives of $\mathcal{L}_1$: $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$

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More English connectives

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- 'It must be the case that'
- 'Pope Benedict XVI thought that'
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Only some English connectives can be captured in $\mathcal{L}_1$.
None of these connectives can be.

Truth functionality

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Consider the false sentences A_1 and A_2

A_1 V. Halbach is giving this lecture.

A_2 Two plus two equals five.

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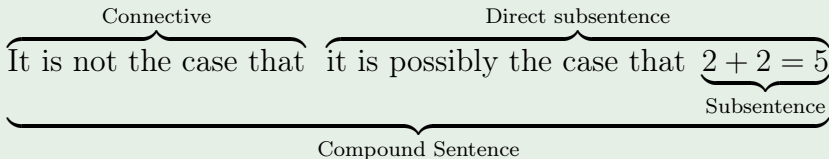
It is possibly the case that A_2 . F

Characterisation: truth-functional (p. 54)

A connective is **truth-functional** if and only if the truth-value of the compound sentence cannot be changed by replacing a direct subsentence with another having the same truth-value.

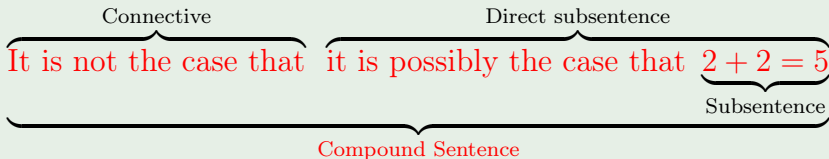
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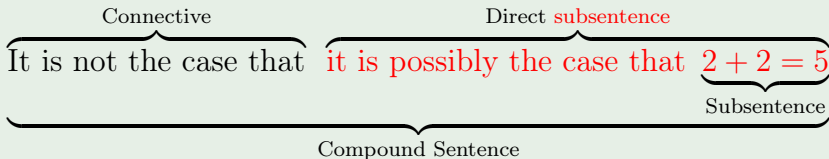
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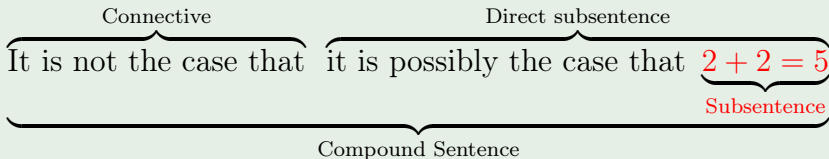
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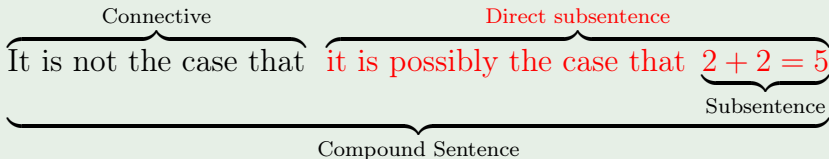
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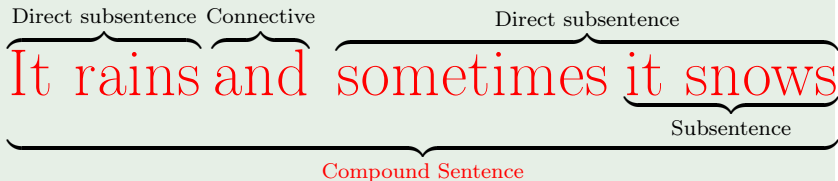
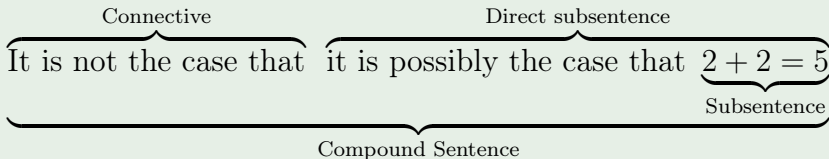
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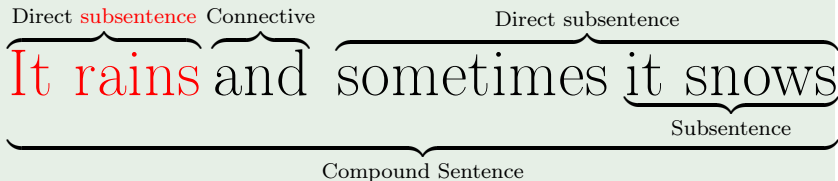
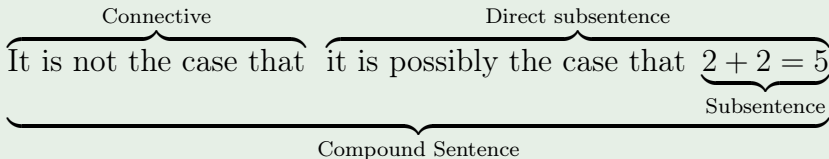
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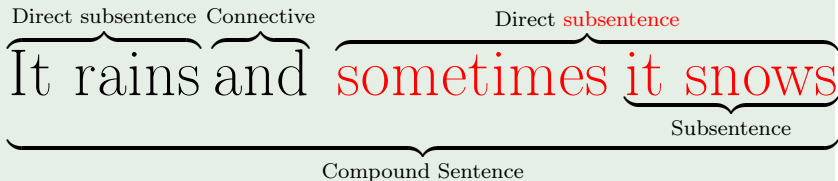
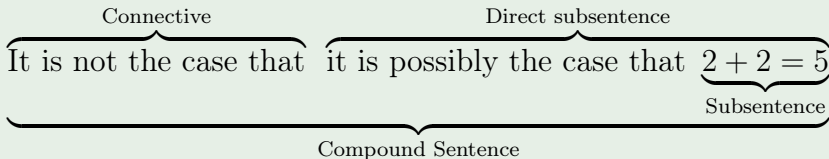
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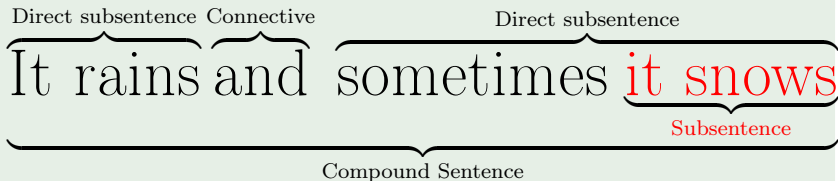
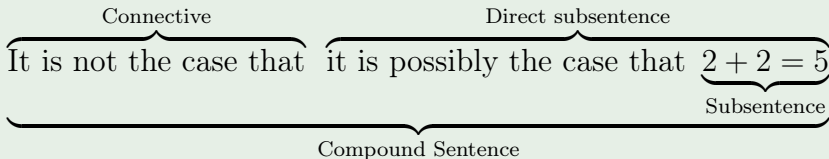
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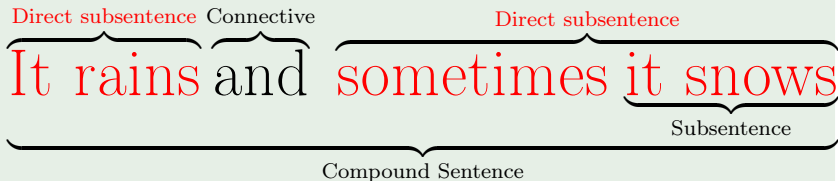
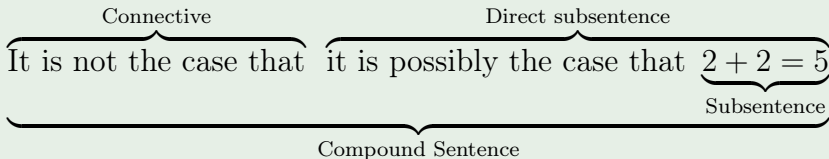
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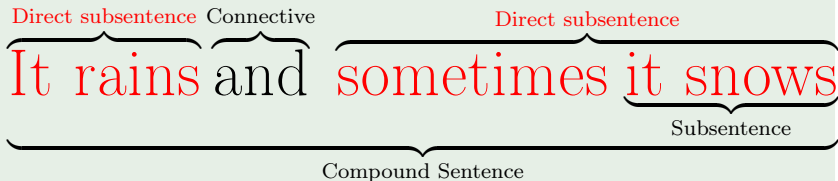
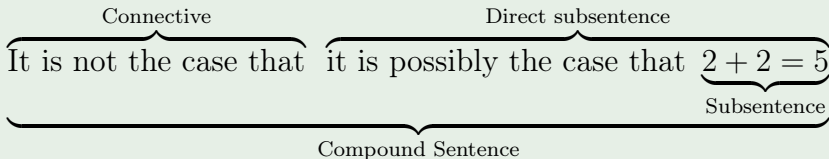
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NB: replacing non-direct subsentences may change the truth-value.

Formalisation

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Formalisation

$\neg R$

Dictionary

R : Russell likes logic.

Formalise:

Russell likes logic and philosophers like conceptual analysis.

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Formalisation
 $(R \wedge P)$

Dictionary

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Note: it's fine to use letters other than P, Q, R when formalising English sentences.

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Formalisation: $\neg R$

Dictionary: R : Russell likes logic.

Formalise:

Neither Russell nor Whitehead likes logic.

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Paraphrase: It is not the case that Russell likes logic.

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Paraphrase: It is not the case that Russell likes logic.

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Dictionary: R : Russell likes logic. W : Whitehead likes logic.

Common variants

Here are some of the most common variants of the standard connectives.

$\mathcal{L}_1$	standard connective	some other formulations
$\wedge$	and	but, although
$\vee$	or	unless
$\neg$	it is not the case that	not, none, never
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Rules of thumb for $\rightarrow$

Formalise:

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Rules of thumb for $\rightarrow$

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Formalise:

- (1) If John revised, [then] he passed. $R \rightarrow P$
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'If' mid-sentence corresponds to ' $\leftarrow$ '; 'only if' to $\rightarrow$.

Differences between $\rightarrow$ and 'if'

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Formalise

If the lecturer hadn't shown up last week, Plato would have given the lecture.

Differences between $\rightarrow$ and 'if'

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If the lecturer hadn't shown up last week, Plato would have given the lecture.

Consider: $\neg S \rightarrow P$.

Dictionary: S : The lecturer showed up last week.

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The English sentence appears to be false.

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P : Plato gave the lecture.

The English sentence appears to be false.

But when $|S|_{\mathcal{A}} = \text{T}$, $|\neg S \rightarrow P|_{\mathcal{A}} = \text{T}$.

Differences between $\rightarrow$ and 'if'

Formalise

If the lecturer hadn't shown up last week, Plato would have given the lecture.

Consider: $\neg S \rightarrow P$.

Dictionary: S : The lecturer showed up last week.

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The English sentence appears to be false.

But when $|S|_{\mathcal{A}} = \text{T}$, $|\neg S \rightarrow P|_{\mathcal{A}} = \text{T}$.

See Sainsbury, *Logical Forms*, ch. 2 for further discussion.

Complex Sentences

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Formalise

If David folded or David didn't have the ace, Victoria won.

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If David folded or David didn't have the ace, Victoria won.

Paraphrase

If David folded or David didn't have the ace, Victoria won.

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(If David folded or David didn't have the ace, Victoria won)

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If David folded or David didn't have the ace, Victoria won.

Paraphrase

(If ((David folded) or it is not the case that David had the ace), Victoria won)

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Complex Sentences

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If David folded or David didn't have the ace, Victoria won.

Paraphrase

(If ((David folded) or it is not the case that (David had the ace)), **Victoria won**)

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If David folded or David didn't have the ace, Victoria won.

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Complex Sentences

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Logical Form

(If ((David folded) or it is not the case that (David had the ace)), (Victoria won))

This is in (propositional) [logical form](#).

- All connectives are standard connectives
- No sentence can be further formalised in $\mathcal{L}_1$.

Complex Sentences

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If David folded or David didn't have the ace, Victoria won.

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Formalisation: $((F \vee \neg A) \rightarrow W)$

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A : David had the ace.

W : Victoria won.

Sometimes the paraphrase may need to be quite loose.

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Formalise

- (1) Exactly one of the following happened: David won or Victoria won.

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Formalisation: $(D \wedge \neg V) \vee (V \wedge \neg D)$

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$(D \wedge \neg V \wedge \neg T) \vee (V \wedge \neg D \wedge \neg T) \vee (T \wedge \neg D \wedge \neg V)$

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Dictionary: T: It was a tie.

(and as before)

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Example

David's hand was weak and Victoria was bound to win unless the Jack came up on the turn.

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The **scope** of an occurrence of a connective in a sentence ϕ is the occurrence of the smallest subsentence of ϕ that contains this occurrence of the connective.

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In (1): $\vee$ has wider scope.

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Scope of $\vee$

$$(2) D \wedge \underbrace{(V \vee J)}_{\text{Scope of } \wedge}$$

Scope of $\vee$

In (1): $\vee$ has wider scope.

In (2): $\wedge$ has wider scope.

More on paraphrase

- (1) Tom and Jerry are animals.
- (2) Tom and Jerry are apart.
- (3) Jerry is a white mouse.
- (4) Jerry is a large mouse.

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Worked example: are these acceptable paraphrases?

- (1) Tom is an animal and Jerry is an animal.
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Logical notions in English

Recall last week's definitions of tautology, contradiction and validity for $\mathcal{L}_1$ sentences and arguments.

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Definition

- (1) An English sentence is a **tautology** if and only if its formalisation in propositional logic is a tautology.
- (2) An English sentence is a **propositional contradiction** if and only if its formalisation in propositional logic is a contradiction.
- (3) An argument in English is **propositionally valid** if and only if its formalisation in $\mathcal{L}_1$ is valid.

Worked Example

Show that the following argument is propositionally valid.

Unless CO₂-emissions are cut, there will be more floods.
CO₂-emissions won't be cut. Therefore there will be more floods.

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C: CO₂ emissions will be cut.

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Dictionary.

C : CO₂ emissions will be cut.

M : There will be more floods.

- Next, formalise the premisses and conclusion.
- Finally, check the formalised argument is valid.

Worked example (cont.)

P1 Unless CO₂-emissions are cut, there will be more floods.

P2 CO₂-emissions won't be cut.

C There will be more floods

Worked example (cont.)

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Formalise the argument:

P1 Paraphrase:

CO₂-emissions will be cut or there will be more floods.

Worked example (cont.)

P1 Unless CO₂-emissions are cut, there will be more floods.

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C There will be more floods

Formalise the argument:

P1 Paraphrase:

CO₂-emissions will be cut or there will be more floods.

Formalisation: $C \vee M$.

Worked example (cont.)

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Formalise the argument:

P1 Paraphrase:

CO₂-emissions will be cut or there will be more floods.

Formalisation: $C \vee M$.

P2 Paraphrase:

It's not the case that CO₂-emissions will be cut.

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Formalise the argument:

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Formalisation: $\neg C$.

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Formalisation: $C \vee M$.

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Formalisation: $\neg C$.

C Formalisation: M .

It remains to show.

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You know two ways to do this.

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You know two ways to do this.

- Method 1: Forwards truth table.
- Method 2: Backwards truth table.

Backwards truth-table

C	M	$(C \vee M)$	$\neg C$	M

It remains to show.

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You know two ways to do this.

- Method 1: Forwards truth table.
- Method 2: Backwards truth table.

Backwards truth-table

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It remains to show.

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So, the English argument is propositionally valid.

<http://logicmanual.philosophy.ox.ac.uk>