INTRODUCTION TO LOGIC

Lecture 2

Syntax and Semantics of Propositional Logic.

Dr. James Studd

Logic is the beginning of wisdom.
Thomas Aquinas

Outline

1. Syntax vs Semantics.
2. Syntax of $\mathcal{L}_1$.
3. Semantics of $\mathcal{L}_1$.
4. Truth-table methods.

Syntax

Syntax is all about **expressions**: words and sentences.

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- ‘Bertrand Russell likes logic’ is a sentence.

Syntax

Syntax is all about **expressions**: words and sentences.

Examples of syntactic claims

- ‘Bertrand Russell’ is a proper noun.
- ‘likes logic’ is a verb phrase.
- ‘Bertrand Russell likes logic’ is a sentence.
- Combining a proper noun and a verb phrase in this way makes a sentence.

Semantics

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- 'Bertrand Russell' refers to Bertrand Russell.
- 'likes logic' expresses a property Russell has.

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Semantics is all about **meanings** of expressions.

Examples of semantic claims

- ‘Bertrand Russell’ refers to a British philosopher.
- ‘Bertrand Russell’ refers to Bertrand Russell.
- ‘likes logic’ expresses a property Russell has.
- ‘Bertrand Russell likes logic’ is true.

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Mention

- The first occurrence of 'Bertrand Russell' is an example of mention.
- This occurrence (with quotes) refers to an expression.

Use

- The second occurrence of 'Bertrand Russell' is an example of use.
- This occurrence (without quotes) refers to a man.

Syntax: English vs. $\mathcal{L}_1$.

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Some basic expressions of $\mathcal{L}_1$

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Some basic expressions of $\mathcal{L}_1$

(1) **Sentence letters**: e.g. ‘P’, ‘Q’.

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Combining sentences and connectives makes new sentences.

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- ‘$\neg$’ and ‘P’ make: ‘$\neg P$’.

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- '$\neg$' and 'P' make: '$\neg P$'.
- 'Bertrand Russell likes logic' and 'and' and 'Philosophers like conceptual analysis' make:
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- 'P', '$\wedge$' and 'Q' make: '$(P \wedge Q)$'.

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Here's the full list of $\mathcal{L}_1$-connectives.

name	in English	symbol
conjunction	and	$\wedge$
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negation	it is not the case that	$\neg$
arrow	if . . . then	$\to$
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Greek letters: ϕ ('PHI') and ψ ('PSI'): not part of $\mathcal{L}_1$.

How to build a sentence of $\mathcal{L}_1$

Example

The following is a sentence of $\mathcal{L}_1$:

Definition of $\mathcal{L}_1$-sentences (repeated from previous page)

(i) All sentence letters are sentences of $\mathcal{L}_1$.

(ii) If ϕ and ψ are sentences of $\mathcal{L}_1$, then $\neg\phi$, $(\phi \land \psi)$, $(\phi \lor \psi)$, $(\phi \to \psi)$ and $(\phi \leftrightarrow \psi)$ are sentences of $\mathcal{L}_1$.

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How to build a sentence of $\mathcal{L}_1$

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How to build a sentence of $\mathcal{L}_1$

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$(P \wedge Q)$ R_{45}

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The following is a sentence of $\mathcal{L}_1$:

$(P \land Q) \qquad \neg R_{45}$

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How to build a sentence of $\mathcal{L}_1$

Example

The following is a sentence of $\mathcal{L}_1$:

$$((P \land Q) \rightarrow (P \lor \neg R_{45}))$$

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How to build a sentence of $\mathcal{L}_1$

Example

The following is a sentence of $\mathcal{L}_1$:

$$((P \wedge Q) \rightarrow (P \vee \neg R_{45})) \qquad\qquad P_3$$

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How to build a sentence of $\mathcal{L}_1$

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The following is a sentence of $\mathcal{L}_1$:

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How to build a sentence of $\mathcal{L}_1$

Example

The following is a sentence of $\mathcal{L}_1$:

$$((P \wedge Q) \to (P \vee \neg R_{45})) \qquad (P_3 \vee R)$$

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The following is a sentence of $\mathcal{L}_1$:

$$(((P \land Q) \to (P \lor \neg R_{45})) \leftrightarrow \neg((P_3 \lor R) \lor R))$$

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The following is a sentence of $\mathcal{L}_1$:

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Object vs. Metalanguage

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- Subject specific expressions: $P, Q, R, \ldots$
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We can think of an $\mathcal{L}_1$-structure as an infinite list that provides a value T or F for every sentence letter.

	P	Q	R	P_1	Q_1	R_1	P_2	Q_2	R_2	$\cdots$
$\mathcal{A}:$	T	F	F	F	T	F	T	T	F	$\cdots$

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We use $\mathcal{A}$, $\mathcal{B}$, etc. to stand for $\mathcal{L}_1$-structures.

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The meaning of $\neg$ is summarised in its **truth table**.

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In words: $|\neg\phi|_{\mathcal{A}} = \mathrm{T}$ if and only if $|\phi|_{\mathcal{A}} = \mathrm{F}$.

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Compute:

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Truth-values of complex sentences 2/3

Truth-conditions for $\wedge$ and $\vee$

The meanings of $\wedge$ and $\vee$ are given by the truth tables:

ϕ	ψ	$(\phi \wedge \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

ϕ	ψ	$(\phi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

Truth-values of complex sentences 2/3

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T	T	T
T	F	F
F	T	F
F	F	F

ϕ	ψ	$(\phi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

Truth-values of complex sentences 2/3

Truth-conditions for ∧ and ∨

The meanings of ∧ and ∨ are given by the truth tables:

ϕ	ψ	$(\phi \wedge \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

ϕ	ψ	$(\phi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

$|(\phi \wedge \psi)|_{\mathcal{A}} = \mathrm{T}$ if and only if $|\phi|_{\mathcal{A}} = \mathrm{T}$ and $|\psi|_{\mathcal{A}} = \mathrm{T}$.

Truth-values of complex sentences 2/3

Truth-conditions for $\wedge$ and $\vee$

The meanings of $\wedge$ and $\vee$ are given by the truth tables:

ϕ	ψ	$(\phi \wedge \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

ϕ	ψ	$(\phi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

$|(\phi \wedge \psi)|_{\mathcal{A}} = \text{T}$ if and only if $|\phi|_{\mathcal{A}} = \text{T}$ and $|\psi|_{\mathcal{A}} = \text{T}$.

Truth-values of complex sentences 2/3

Truth-conditions for ∧ and ∨

The meanings of ∧ and ∨ are given by the truth tables:

ϕ	ψ	$(\phi \wedge \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

ϕ	ψ	$(\phi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

$|(\phi \wedge \psi)|_{\mathcal{A}} = \text{T}$ if and only if $|\phi|_{\mathcal{A}} = \text{T}$ and $|\psi|_{\mathcal{A}} = \text{T}$.
$|(\phi \vee \psi)|_{\mathcal{A}} = \text{T}$ if and only if $|\phi|_{\mathcal{A}} = \text{T}$ or $|\psi|_{\mathcal{A}} = \text{T}$ (or both).

Truth-values of complex sentences 3/3

Truth-conditions for $\rightarrow$ and $\leftrightarrow$

The meanings of $\rightarrow$ and $\leftrightarrow$ are given by the truth tables:

ϕ	ψ	$(\phi \rightarrow \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Truth-values of complex sentences 3/3

Truth-conditions for $\to$ and $\leftrightarrow$

The meanings of $\to$ and $\leftrightarrow$ are given by the truth tables:

ϕ	ψ	$(\phi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Truth-values of complex sentences 3/3

Truth-conditions for $\to$ and $\leftrightarrow$

The meanings of $\to$ and $\leftrightarrow$ are given by the truth tables:

ϕ	ψ	$(\phi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

$|(\phi \to \psi)|_{\mathcal{A}} = \mathrm{T}$ if and only if $|\phi|_{\mathcal{A}} = \mathrm{F}$ or $|\psi|_{\mathcal{A}} = \mathrm{T}$.

Truth-values of complex sentences 3/3

Truth-conditions for $\to$ and $\leftrightarrow$

The meanings of $\to$ and $\leftrightarrow$ are given by the truth tables:

ϕ	ψ	$(\phi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

$|(\phi \to \psi)|_{\mathcal{A}} = \mathrm{T}$ if and only if $|\phi|_{\mathcal{A}} = \mathrm{F}$ or $|\psi|_{\mathcal{A}} = \mathrm{T}$.

Truth-values of complex sentences 3/3

Truth-conditions for $\rightarrow$ and $\leftrightarrow$

The meanings of $\rightarrow$ and $\leftrightarrow$ are given by the truth tables:

ϕ	ψ	$(\phi \rightarrow \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

$|(\phi \rightarrow \psi)|_{\mathcal{A}} = \text{T}$ if and only if $|\phi|_{\mathcal{A}} = \text{F}$ or $|\psi|_{\mathcal{A}} = \text{T}$.
$|(\phi \leftrightarrow \psi)|_{\mathcal{A}} = \text{T}$ if and only if $|\phi|_{\mathcal{A}} = |\psi|_{\mathcal{A}}$.

Worked example 2

Let $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \land Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \land Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \text{F}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \mathrm{F}$ and $|(P \wedge Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \land Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \text{F}$ and $|(P \land Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \mathrm{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \mathrm{F}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \text{F}$
2. $|\neg(P \to Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \land Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \text{F}$ and $|(P \land Q)|_{\mathcal{B}} = \text{F}$
2. $|\neg(P \to Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \land Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \text{F}$ and $|(P \land Q)|_{\mathcal{B}} = \text{F}$
2. $|\neg(P \to Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \land Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \mathrm{F}$ and $|(P \land Q)|_{\mathcal{B}} = \mathrm{F}$
2. $|\neg(P \to Q)|_{\mathcal{B}} = \mathrm{T}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \land Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \mathrm{F}$ and $|(P \land Q)|_{\mathcal{B}} = \mathrm{F}$
2. $|\neg(P \to Q)|_{\mathcal{B}} = \mathrm{T}$
3. $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \mathrm{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \mathrm{F}$
2. $|\neg(P \to Q)|_{\mathcal{B}} = \mathrm{T}$
3. $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_\mathcal{B} = \text{T}$ and $|Q|_\mathcal{B} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_\mathcal{B}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_\mathcal{B} = \text{F}$ and $|(P \wedge Q)|_\mathcal{B} = \text{F}$
2. $|\neg(P \to Q)|_\mathcal{B} = \text{T}$
3. $|\neg(P \to Q) \to (P \wedge Q)|_\mathcal{B}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \mathrm{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \mathrm{F}$
2. $|\neg(P \to Q)|_{\mathcal{B}} = \mathrm{T}$
3. $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}} = \mathrm{F}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \land Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \text{F}$ and $|(P \land Q)|_{\mathcal{B}} = \text{F}$
2. $|\neg(P \to Q)|_{\mathcal{B}} = \text{T}$
3. $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}} = \text{F}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P \land Q)$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

P	Q	$\neg\ (P\ \to Q)\ \to (P \land Q)$
T	F	

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

P	Q	$\neg$	$($	P	$\to$	$Q)$	$\to$	$(P \land Q)$
T	F			T				

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P \wedge Q)$
T	F		T		F		

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	F		T		F		T		

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\land$	$Q)$
T	F		T		F		T		F

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	F	T		F		T		F	

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\land$	$Q)$
T	F		T		F		T	F	F

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\land$	$Q)$
T	F	T		F		T		F	F

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	F		T	F	F		T	F	F

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\land$	$Q)$
T	F		T	F	F		T	F	F

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	F	T	T	F	F		T	F	F

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	F	T	T	F	F		T	F	F

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	F	T	T	F	F	**F**	T	F	F

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
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P	Q	$\neg\,(P \to Q) \to (P \wedge Q)$
T	T	
T	F	
F	T	
F	F	

Using the same technique we can fill out the full truth table for $\neg(P \to Q) \to (P \land Q)$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\land$	$Q)$
T	T		T						
T	F								
F	T								
F	F								

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T	T	T
T	F	T
F	T	
F	F	

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P	Q	$\neg\,(P \to Q) \to (P \land Q)$
T	T	T
T	F	T
F	T	F
F	F	

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P	Q	$\neg\ (P \to Q) \to (P \wedge Q)$
T	T	T
T	F	T
F	T	F
F	F	F

Using the same technique we can fill out the full truth table for $\neg(P \to Q) \to (P \land Q)$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\land$	$Q)$
T	T		T				T		
T	F		T						
F	T		F						
F	F		F						

Using the same technique we can fill out the full truth table for $\neg(P \to Q) \to (P \wedge Q)$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	T		T				T		
T	F		T				T		
F	T		F						
F	F		F						

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P	Q	$\neg$	$(P \to Q)$	$\to$	$(P \wedge Q)$
T	T	T		T	
T	F	T		T	
F	T	F		F	
F	F	F			

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T	T	T		T	
T	F	T		T	
F	T	F		F	
F	F	F		F	

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T	T		T		T		T		
T	F		T				T		
F	T		F				F		
F	F		F				F		

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T	T		T		T		T		
T	F		T		F		T		
F	T		F				F		
F	F		F				F		

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P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\land$	$Q)$
T	T		T		T		T		
T	F		T		F		T		
F	T		F		T		F		
F	F		F				F		

Using the same technique we can fill out the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg$	$(P$	$\rightarrow$	$Q)$	$\rightarrow$	$(P$	$\wedge$	$Q)$
T	T		T		T		T		
T	F		T		F		T		
F	T		F		T		F		
F	F		F		F		F		

Using the same technique we can fill out the full truth table for $\neg(P \to Q) \to (P \wedge Q)$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	T		T		T		T		T
T	F		T		F		T		
F	T		F		T		F		
F	F		F		F		F		

Using the same technique we can fill out the full truth table for $\neg(P \to Q) \to (P \land Q)$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\land$	$Q)$
T	T		T		T		T		T
T	F		T		F		T		F
F	T		F		T		F		
F	F		F		F		F		

Using the same technique we can fill out the full truth table for $\neg(P \to Q) \to (P \wedge Q)$

P	Q	$\neg\,(P$	$\to Q)$	$\to (P$	$\wedge\, Q)$
T	T	T	T	T	T
T	F	T	F	T	F
F	T	F	T	F	T
F	F	F	F	F	

Using the same technique we can fill out the full truth table for $\neg(P \to Q) \to (P \wedge Q)$

P	Q	$\neg$	$(P \to Q)$	$\to$	$(P \wedge Q)$
T	T	T	T	T	T
T	F	T	F	T	F
F	T	F	T	F	T
F	F	F	F	F	F

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P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	T		T	T	T		T		T
T	F		T		F		T		F
F	T		F		T		F		T
F	F		F		F		F		F

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

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T	T		T	T	T		T		T
T	F		T	F	F		T		F
F	T		F		T		F		T
F	F		F		F		F		F

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T	F
F	T

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T	T		T	T	T		T		T
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P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\land$	$Q)$
T	T		T	T	T		T		T
T	F		T	F	F		T		F
F	T		F	T	T		F		T
F	F		F	T	F		F		F

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T	T		T	T	T		T	T	T
T	F		T	F	F		T	F	F
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F	T	F	F	T	T		F	F	T
F	F		F	T	F		F	F	F

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F	T	F	F	T	T		F	F	T
F	F	F	F	T	F		F	F	F

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T	T	F	T	T	T	**T**	T	T	T
T	F	T	T	F	F		T	F	F
F	T	F	F	T	T		F	F	T
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T	F	T	T	F	F	**F**	T	F	F
F	T	F	F	T	T		F	F	T
F	F	F	F	T	F		F	F	F

ϕ	$\neg\phi$
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T	T	T	T
T	F	F	F
F	T	F	T
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T	T	F	T	T	T	**T**	T	T	T
T	F	T	T	F	F	**F**	T	F	F
F	T	F	F	T	T	**T**	F	F	T
F	F	F	F	T	F		F	F	F

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T	T	F	T	T	T	**T**	T	T	T
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F	T	F	F	T	T	**T**	F	F	T
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F	F	F	T

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P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	T	F	T	T	T	**<u>T</u>**	T	T	T
T	F	T	T	F	F	**<u>F</u>**	T	F	F
F	T	F	F	T	T	**<u>T</u>**	F	F	T
F	F	F	F	T	F	**<u>T</u>**	F	F	F

The main column (underlined) gives the truth-value of the whole sentence.

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
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Validity

Let Γ be a set of sentences of $\mathcal{L}_1$ and ϕ a sentence of $\mathcal{L}_1$.

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Definition

The argument with all sentences in Γ as premisses and ϕ as conclusion is **valid** if and only if there is no $\mathcal{L}_1$-structure under which:

(i) all sentences in Γ are true; and

(ii) ϕ is false.

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Notation: when this argument is valid we write $\Gamma \vDash \phi$.

$\{P \to \neg Q, Q\} \models \neg P$ means that the argument whose premises are $P \to \neg Q$ and Q, and whose conclusion is $\neg P$ is valid.
Also written: $P \to \neg Q, Q \models \neg P$

Worked example 3

We can use truth-tables to show that $\mathcal{L}_1$-arguments are valid.

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P	Q	P	$\to$	$\neg$	Q	Q	$\neg$	P
T	T	T	**F**	F	T	**T**	**F**	T
T	F	T	**T**	T	F	**F**	**F**	T
F	T	F	**T**	F	T	**T**	**T**	F
F	F	F	**T**	T	F	**F**	**T**	F

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Show that $\{P \to \neg Q, Q\} \models \neg P$.

P	Q	P	$\to$	$\neg$	Q	Q	$\neg$	P
T	T	T	**F**	F	T	**T**	**F**	T
T	F	T	**T**	T	F	**F**	**F**	T
F	T	F	**T**	F	T	**T**	**T**	F
F	F	F	**T**	T	F	**F**	**T**	F

Rows correspond to interpretations.

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Show that $\{P \to \neg Q, Q\} \models \neg P$.

P	Q	$P \to \neg Q$				Q	$\neg P$	
T	T	T	**F**	F	T	**T**	**F**	T
T	F	T	**T**	T	F	**F**	**F**	T
F	T	F	**T**	F	T	**T**	**T**	F
F	F	F	**T**	T	F	**F**	**T**	F

Rows correspond to interpretations.
One needs to check that there is no row in which all the premisses are assigned T and the conclusion is assigned F.

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Show that $\{P \to \neg Q, Q\} \models \neg P$.

	P	Q	P	$\to$	$\neg$	Q	Q	$\neg$	P
▶	T	T	T	**F**	F	T	**T**	**F**	T
	T	F	T	**T**	T	F	**F**	**F**	T
	F	T	F	**T**	F	T	**T**	**T**	F
	F	F	F	**T**	T	F	**F**	**T**	F

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	P	Q	$P \to \neg Q$	Q	$\neg P$
	T	T	T **F** F T	**T**	**F** T
►	T	F	T **T** T F	**F**	**F** T
	F	T	F **T** F T	**T**	**T** F
	F	F	F **T** T F	**F**	**T** F

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Show that $\{P \to \neg Q, Q\} \models \neg P$.

	P	Q	P	$\to$	$\neg$	Q	Q	$\neg$	P
	T	T	T	**F**	F	T	**T**	**F**	T
	T	F	T	**T**	T	F	**F**	**F**	T
►	F	T	F	**T**	F	T	**T**	**T**	F
	F	F	F	**T**	T	F	**F**	**T**	F

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	P	Q	P	$\to$	$\neg$	Q	Q	$\neg$	P
	T	T	T	**F**	F	T	**T**	**F**	T
	T	F	T	**T**	T	F	**F**	**F**	T
	F	T	F	**T**	F	T	**T**	**T**	F
►	F	F	F	**T**	T	F	**F**	**T**	F

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Other logical notions

Definition

A sentence ϕ of $\mathcal{L}_1$ is **logically true** (a **tautology**) iff:

- ϕ is true under all $\mathcal{L}_1$-structures.

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A sentence ϕ of $\mathcal{L}_1$ is **logically true** (a **tautology**) iff:

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e.g. $P \vee \neg P$, and $P \to P$ are tautologies.

Truth tables of tautologies

Every row in the main column is a T.

P	P	$\vee$	$\neg$	P	P	$\to$	P
T	T	**T**	F	T	T	**T**	T
F	F	**T**	T	F	F	**T**	F

Definition

A sentence ϕ of $\mathcal{L}_1$ is a **contradiction** iff:

- ϕ is not true under any $\mathcal{L}_1$-structure.

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e.g. $P \wedge \neg P$, and $\neg(P \to P)$ are contradictions.

Truth tables of contradictions

Every row in the main column is an F.

P	P	$\wedge$	$\neg$	P	$\neg$	$(P$	$\to$	$P)$
T	T	**F**	F	T	**F**	T	T	T
F	F	**F**	T	F	**F**	F	T	F

Definition

Sentences ϕ and ψ are **logically equivalent** iff:

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- $P \wedge Q$ and $\neg(\neg P \vee \neg Q)$ are logically equivalent.

Truth tables of logical equivalents

The truth-values in the main columns agree.

P	Q	P	$\wedge$	Q	$\neg$	$(\neg$	P	$\vee$	$\neg$	$Q)$
T	T	T	**T**	T	**T**	F	T	F	F	T
T	F	T	**F**	F	**F**	F	T	T	T	F
F	T	F	**F**	T	**F**	T	F	T	F	T
F	F	F	**F**	F	**F**	T	F	T	T	F

Worked example 4

Example

Show that the sentence $(P \to (\neg Q \land R)) \lor P$ is a tautology.

Worked example 4

Example

Show that the sentence $(P \to (\neg Q \wedge R)) \vee P$ is a tautology.

Method 1: Full truth table

- Write out the truth table for $(P \to (\neg Q \wedge R)) \vee P$.
- Check there's a T in every row of the main column.

Worked example 4

Example

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Method 1: Full truth table

- Write out the truth table for $(P \to (\neg Q \wedge R)) \vee P$.
- Check there's a T in every row of the main column.

P	Q	R	$(P$	$\to$	$(\neg$	Q	$\wedge$	$R))$	$\vee$	P
T	T	T	T	F	F	T	F	T	**T**	T
T	T	F	T	F	F	T	F	F	**T**	T
T	F	T	T	T	T	F	T	T	**T**	T
T	F	F	T	F	T	F	F	F	**T**	T
F	T	T	F	T	F	T	F	T	**T**	F
F	T	F	F	T	F	T	F	F	**T**	F
F	F	T	F	T	T	F	T	T	**T**	F
F	F	F	F	T	T	F	F	F	**T**	F

Worked example 4 (cont.)

Show that the sentence $(P \to (\neg Q \wedge R)) \vee P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P$	$\to$	$(\neg$	Q	$\wedge$	$R))$	$\vee$	P

Worked example 4 (cont.)

Show that the sentence $(P \to (\neg Q \wedge R)) \vee P$ is a tautology.

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									F	

Worked example 4 (cont.)

Show that the sentence $(P \to (\neg Q \land R)) \lor P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P$	$\to$	$(\neg$	Q	$\land$	$R))$	$\lor$	P
									F	

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \lor \psi)$	$(\phi \to \psi)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Worked example 4 (cont.)

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Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P$	$\to$	$(\neg$	Q	$\wedge$	$R))$	$\vee$	P
				F_1					F	

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \vee \psi)$	$(\phi \to \psi)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Worked example 4 (cont.)

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Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P$	$\to$	$(\neg$	Q	$\wedge$	$R))$	$\vee$	P
				F_1					F	F_2

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \vee \psi)$	$(\phi \to \psi)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Worked example 4 (cont.)

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P	Q	R	$(P$	$\to$	$(\neg$	Q	$\wedge$	$R))$	$\vee$	P
				F_1					F	F_2

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \vee \psi)$	$(\phi \to \psi)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Worked example 4 (cont.)

Show that the sentence $(P \to (\neg Q \land R)) \lor P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P$	$\to$	$(\neg$	Q	$\land$	$R))$	$\lor$	P
			T_3	F_1					F	F_2

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \lor \psi)$	$(\phi \to \psi)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Worked example 4 (cont.)

Show that the sentence $(P \to (\neg Q \land R)) \lor P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P$	$\to$	$(\neg\, Q \land R))$	$\lor$	P
			T_3	F_1		F	F_2

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \lor \psi)$	$(\phi \to \psi)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Worked example 4 (cont.)

Show that the sentence $(P \to (\neg Q \wedge R)) \vee P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P$	$\to$	$(\neg$	Q	$\wedge$	$R))$	$\vee$	P
			?	F_1					F	F_2

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \vee \psi)$	$(\phi \to \psi)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Worked example 5

Example

Show that $P \leftrightarrow \neg Q \vDash \neg(P \leftrightarrow Q)$

Worked example 5

Example

Show that $P \leftrightarrow \neg Q \vDash \neg(P \leftrightarrow Q)$

Method 1: Full truth table

- Write out the full truth table.
- Check there's no row in which the main column of the premiss is T and the main column of the conclusion is F.

Worked example 5 (cont.)

Show that $P \leftrightarrow \neg Q \models \neg(P \leftrightarrow Q)$

Worked example 5 (cont.)

Show that $P \leftrightarrow \neg Q \models \neg(P \leftrightarrow Q)$

Method 2: Backwards truth table

- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction.

x

Worked example 5 (cont.)

Show that $P \leftrightarrow \neg Q \vDash \neg(P \leftrightarrow Q)$

Method 2: Backwards truth table

- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction.

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	$(P$	$\leftrightarrow$	$Q)$

Worked example 5 (cont.)

Show that $P \leftrightarrow \neg Q \models \neg(P \leftrightarrow Q)$

Method 2: Backwards truth table

- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction.

x

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	$(P$	$\leftrightarrow$	$Q)$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Worked example 5 (cont.)

Show that $P \leftrightarrow \neg Q \vDash \neg(P \leftrightarrow Q)$

Method 2: Backwards truth table

- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction.

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	$(P$	$\leftrightarrow$	$Q)$
			T						

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Worked example 5 (cont.)

Show that $P \leftrightarrow \neg Q \vDash \neg(P \leftrightarrow Q)$

Method 2: Backwards truth table

- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction.

x

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	$(P$	$\leftrightarrow$	$Q)$
			T			F			

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
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Worked example 5 (cont.)

Show that $P \leftrightarrow \neg Q \vDash \neg(P \leftrightarrow Q)$

Method 2: Backwards truth table

- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction. x

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	$($ P	$\leftrightarrow$	$Q)$
			T			F			

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Worked example 5 (cont.)

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Method 2: Backwards truth table

- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction.

x

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	$(P$	$\leftrightarrow$	$Q)$
			T			F		T_1	

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Worked example 5 (cont.)

Show that $P \leftrightarrow \neg Q \models \neg(P \leftrightarrow Q)$

Method 2: Backwards truth table

- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction.

x

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	$(P$	$\leftrightarrow$	$Q)$
			T			F		T_1	

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Worked example 5 (cont.)

Show that $P \leftrightarrow \neg Q \models \neg(P \leftrightarrow Q)$

Method 2: Backwards truth table

- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction. x

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	(P	$\leftrightarrow$	Q)
			T			F		T_1	
			T			F		T_1	

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Worked example 5 (cont.)

Show that $P \leftrightarrow \neg Q \vDash \neg(P \leftrightarrow Q)$

Method 2: Backwards truth table

- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction. x

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	$(P$	$\leftrightarrow$	$Q)$
			T			F	T_2	T_1	
			T			F		T_1	

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Worked example 5 (cont.)

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Method 2: Backwards truth table

- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction.

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	$(P$	$\leftrightarrow$	$Q)$
			T			F	T_2	T_1	T_3
			T			F		T_1	

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Worked example 5 (cont.)

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- Work backwards to obtain a contradiction. x

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	$(P$	$\leftrightarrow$	$Q)$
			T			F	T_2	T_1	T_3
			T			F	F_2	T_1	

ϕ	$\neg\phi$
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ϕ	ψ	$(\phi \leftrightarrow \psi)$
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Worked example 5 (cont.)

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- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction. x

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	$(P$	$\leftrightarrow$	$Q)$
			T			F	T_2	T_1	T_3
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Worked example 5 (cont.)

Show that $P \leftrightarrow \neg Q \vDash \neg(P \leftrightarrow Q)$

Method 2: Backwards truth table

- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction. x

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	$(P$	$\leftrightarrow$	$Q)$
		T_4	T			F	T_2	T_1	T_3
			T			F	F_2	T_1	F_3

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Worked example 5 (cont.)

Show that $P \leftrightarrow \neg Q \models \neg(P \leftrightarrow Q)$

Method 2: Backwards truth table

- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction.

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	$(P$	$\leftrightarrow$	$Q)$
		T_4	T	T_5		F	T_2	T_1	T_3
			T			F	F_2	T_1	F_3

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Worked example 5 (cont.)

Show that $P \leftrightarrow \neg Q \models \neg(P \leftrightarrow Q)$

Method 2: Backwards truth table

- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction.

x

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	$(P$	$\leftrightarrow$	$Q)$
		T_4	T	T_5	?	F	T_2	T_1	T_3
			T			F	F_2	T_1	F_3

ϕ	$\neg\phi$
T	F
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Worked example 5 (cont.)

Show that $P \leftrightarrow \neg Q \models \neg(P \leftrightarrow Q)$

Method 2: Backwards truth table

- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction. x

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	$(P$	$\leftrightarrow$	$Q)$
		T_4	T	T_5	?	F	T_2	T_1	T_3
		F_4	T			F	F_2	T_1	F_3

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Worked example 5 (cont.)

Show that $P \leftrightarrow \neg Q \models \neg(P \leftrightarrow Q)$

Method 2: Backwards truth table

- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction. x

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	$(P$	$\leftrightarrow$	$Q)$
		T_4	T	T_5	?	F	T_2	T_1	T_3
		F_4	T	F_5		F	F_2	T_1	F_3

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
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Worked example 5 (cont.)

Show that $P \leftrightarrow \neg Q \models \neg(P \leftrightarrow Q)$

Method 2: Backwards truth table

- Put a T in the main column of the premiss and an F in the main column of the conclusion.
- Work backwards to obtain a contradiction. x

P	Q	P	$\leftrightarrow$	$\neg$	Q	$\neg$	$(P$	$\leftrightarrow$	$Q)$
		T_4	T	T_5	?	F	T_2	T_1	T_3
		F_4	T	F_5	?	F	F_2	T_1	F_3

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

http://logicmanual.philosophy.ox.ac.uk