# INTRODUCTION TO LOGIC

## Lecture 1

## Validity

## Introduction to Sets and Relations.

Dr. James Studd

Pure logic is the ruin of the spirit.

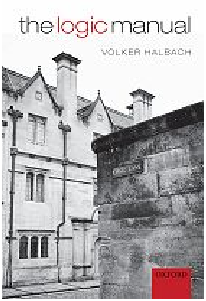
*Antoine de Saint-Exupéry*

# Outline

(1) Introductory
(2) Validity
(3) Course Overview
(4) Sets and Relations

# Resources

- The Logic Manual
- **logicmanual.philosophy.ox.ac.uk**
    - Exercises booklet
    - Lecture slides
    - Worked examples
    - Past examination papers
      some with solutions




- Mark Sainsbury: *Logical Forms: An Introduction to Philosophical Logic*, Blackwell, second edition, 2001, chs. 1–2.

# Why logic?

Logic is the scientific study of valid argument.

- Philosophy is all about arguments and reasoning.
- Logic allows us to rigorously test validity.
- Modern philosophy assumes familiarity with logic.
- Used in linguistics, mathematics, computer science,...
- Helps us make fine-grained conceptual distinctions.
- Logic is compulsory.

# Validity 1/3

**First approximation.**

When an argument is valid, the truth of the premisses **guarantees** the truth of the conclusion. 10

An argument is valid if it '**can't**' be the case that all of the premisses are true and the conclusion is false.

- Validity does **not** depend on contingent facts.
- Validity does **not** depend on laws of nature.
- Validity does **not** depend on the meanings of subject-specific expressions.
- Validity depends purely on the 'form' of the argument.

# Examples

**Argument 1** **Not valid**

Zeno is a tortoise.
Therefore, Zeno is toothless.

The truth of the premiss does not provide a sufficiently strong guarantee of the truth of the conclusion

**Argument 2** **Valid**

Zeno is a tortoise.
All tortoises are toothless.
Therefore, Zeno is toothless.

# Validity 2/3

## Characterisation (p. 19)

An argument is **logically valid** if and only if:
there is no interpretation under which:

**(i)** the premisses are all true, and

**(ii)** the conclusion is false.

# Argument 1 revisited

**Argument 1** **Not valid**

Zeno is a tortoise.
Therefore, Zeno is toothless.

**Argument 1a** **Not valid**

Theresa May is a Conservative.
Therefore, Theresa May is a Liberal Democrat.

There is an interpretation under which:

**(i)** the premiss is true, and
**(ii)** the conclusion is false.

# Argument 2 revisited

**Argument 2** **Valid**

Zeno is a tortoise.
All tortoises are toothless.
Therefore, Zeno is toothless.

**Argument 2a** **Valid**

Theresa May is a Conservative.
All Conservatives are Liberal Democrats.
Therefore, Theresa May is a Liberal Democrat.

**Argument 2b** **Valid**

Radon is a noble gas.
All noble gases are chemical elements.
Therefore, Radon is a chemical element.

Note: argument 2a is a valid argument with a false conclusion.

# Validity 3/3.

**Characterisation (p. 19)**

An argument is **logically valid** if and only if:
there is no [uniform] interpretation [of subject-specific expressions] under which:

**(i)** the premisses are all true, and

**(ii)** the conclusion is false.

- Each occurrence of an expression interpreted in the same way
- Logical expression keep their usual English meanings.

# Subject-specific versus logical expressions

**Examples: logical terms**

all, every, some, no.
not, and, or, unless, if, only if, if and only if.

**Examples: subject-specific terms**

Zeno, Theresa May, France, The North Sea, Radon, soap, bread, GDP, logical positivism, ...
tortoise, toothless, Conservative, nobel gas, philosopher, chemical element, ...
loves, owns, reacts with, voted for, ...

# Argument 2 revisited again

**Argument 2** **Valid**

Zeno is a tortoise.
All tortoises are toothless.
Therefore, Zeno is toothless.

**Argument 3** **Not valid**

Theresa May is a Conservative.
No Conservatives are Liberal Democrats.
Therefore, Theresa May is a Liberal Democrat.

**Argument 4** **Not valid**

Radon is a noble gas.
All noble gases are chemical elements.
Therefore, air is a chemical element.

# Course overview

**1:** Validity; Introduction to Sets and Relations

**2:** Syntax and Semantics of Propositional Logic

**3:** Formalization in Propositional Logic

**4:** The Syntax of Predicate Logic

**5:** The Semantics of Predicate Logic

**6:** Natural Deduction

**7:** Formalization in Predicate Logic

**8:** Identity and Definite Descriptions

# Sets 1/2

## Characterisation

A **set** is a collection of zero or more objects.

- The objects are called **elements** of the set.
- $a \in b$ is short for '$a$ is an element of set $b$'.

## Examples

- The set of positive integers less than 4:
  $\{1, 2, 3\}$ or $\{n : n \text{ is an integer between 1 and 3}\}$
- The set of positive integers:
  $\{1, 2, 3, 4, \ldots\}$ or $\{n : n > 0\}$
- The empty set:
  $\{\ \}$ or $\{x : x \text{ is a round square}\}$ or $\emptyset$

# Sets 2/2

**Fact about sets**

Sets are identical if and only if they have the same elements.

**Example**

The following sets are all identical:

- {Lennon, McCartney, Harrison, Ringo}
- {Ringo, Lennon, Harrison, McCartney}
- {Ringo, Ringo, Ringo, Lennon, Harrison, McCartney}
- $\{x : x \text{ is a Beatle}\}$
- $\{x : x \text{ sang lead vocals on an Abbey Road track}\}$

# Ordered pairs

## Characterisation

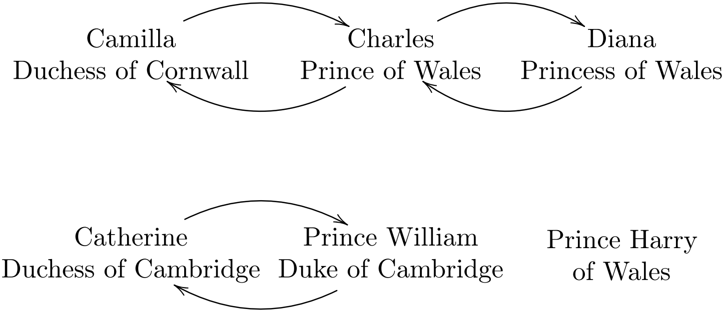
An **ordered pair** comprises two components in a given order.

- $\langle d, e\rangle$ is the ordered pair whose first component is $d$ and whose second component is $e$, **in that order**.

## Example

$\langle \text{London, Munich}\rangle \neq \langle \text{Munich, London}\rangle$
$\{\text{London, Munich}\} = \{\text{Munich, London}\}$

# Relations




**The relation of *having married***

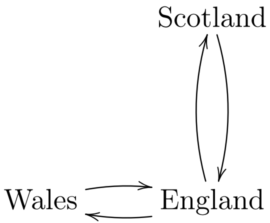
$$\{\langle \text{Charles, Diana}\rangle, \langle \text{Diana, Charles}\rangle,$$
$$\langle \text{Charles, Camilla}\rangle, \langle \text{Camilla, Charles}\rangle,$$
$$\langle \text{Kate, William}\rangle, \langle \text{William, Kate}\rangle, \ldots\}$$

# Worked example

Write down the following relation as a set of ordered pairs.
Draw its arrow diagram. 40

**The relation of *being countries in GB sharing a border***

$$\{\langle \text{England, Scotland}\rangle, \langle \text{Scotland, England}\rangle,$$
$$\langle \text{England, Wales}\rangle, \langle \text{Wales, English}\rangle\}$$



# Relations

## Definition (p. 8)

A set $R$ is a **binary relation** if and only if it contains only ordered pairs.

Informally: $\langle d, e\rangle \in R$ indicates that $d$ stands in $R$ to $e$.

## Example

- The relation of *being a bigger city than.*
  $\{\langle$London, St. Davids$\rangle$, $\langle$London, Brighton$\rangle$, $\langle$Brighton, St. Davids $\rangle$,...$\}$.
  $\{\langle d, e\rangle : d$ is a bigger city than $e\}$.
- The empty set: $\emptyset$

# Properties of relations 1/3

**Definition (p. 9)**

A binary relation $R$ is **reflexive on a set $S$** iff:

- for all $d$ in $S$: the pair $\langle d, d \rangle$ is an element of $R$. 45

Informally: every member of $S$ bears $R$ to itself.

**Example** **Reflexive on the set of human beings**

- The relation of *being the same height as*

**Example** **Not reflexive on this set**

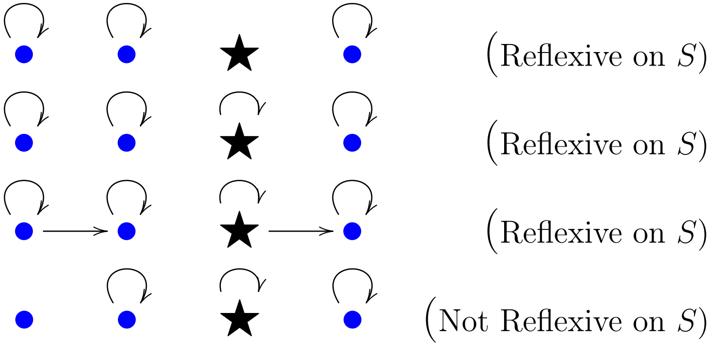
- The relation of *being taller than*

**Example** **Not reflexive on $\{1, 2, 3\}$**

- $\{\langle 1, 1\rangle, \langle 2, 2\rangle, \langle 1, 3\rangle\}$ Reflexive on $\{1, 2\}$

## Reflexivity on $S$

Every point in $S$ has a "loop".




Key: Member of $S$: ●
Non-member of $S$: ★

# Properties of relations 2/3

**Definition (p. 9)**

A binary relation $R$ is **symmetric on set $S$** iff:

- for all $d, e$ in $S$: if $\langle d, e\rangle \in R$ then $\langle e, d\rangle \in R$.

Informally: any member of $S$ bears $R$ to a second only if the second bears $R$ back to the first.

**Example** **Symmetric on the set of human beings**

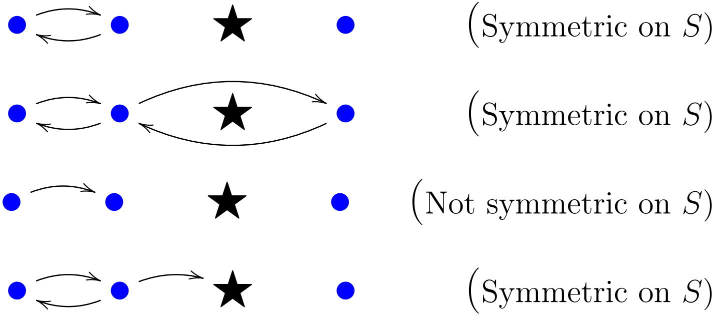
- The relation of *being a sibling of*

**Example** **Not symmetric on this set**

- The relation of *being a brother of*

## Symmetry on $S$

Every "outward route" between points in $S$ has a "return route".



# Properties of relations 3/3

**Definition**

A binary relation $R$ is **transitive on S** iff:

- for all $d, e, f$ in $S$:
  if $\langle d, e\rangle \in R$ and $\langle e, f\rangle \in R$, then also $\langle d, f\rangle \in R$

Informally: if any member of $S$ bears $R$ to a second, and the second also bears $R$ to a third, the first bears $R$ to the third.

**Example** — **Transitive on the set of human beings**

- The relation of *being taller than*

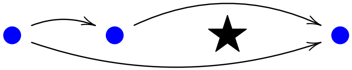
**Example** — **Not transitive on this set**

- The relation of *not having the same height* ($\pm$1cm)

## Transitivity on $S$

Every “double-step” between points in $S$ has a “one-step shortcut”.

(Not transitive on $S$)



(Transitive on $S$)

(Not transitive on $S$)

(Transitive on $S$)

# Functions

## Definition (p. 14)

A binary relation $F$ is a **function** iff for all $d, e, f$:

- if $\langle d, e\rangle \in F$ and $\langle d, f\rangle \in F$ then $e = f$.

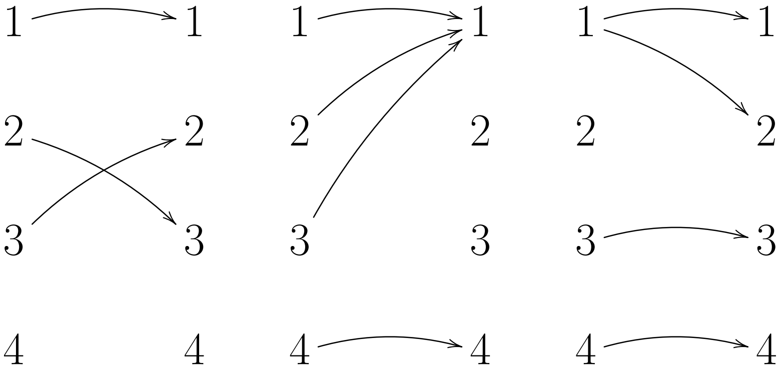
Informally, everything stands in $F$ to at most one thing.

## Example

- The function that squares positive integers.
  $\{\langle 1, 1\rangle, \langle 2, 4\rangle, \langle 3, 9\rangle, \ldots\}$
  $\{\langle x, y\rangle : y = x^2, \text{ for } x \text{ a positive integer}\}$

## $F$ is a function

Everything stands in $F$ to at most one thing ("many-one" or "one-one")




"one-one" function

"many-one" function

"one-many" not a function

# A "straightforward and elementary" example

(a) What is a binary relation?

(b) Consider the relation $R$ of *sharing exactly one parent*:

$$R = \{\langle d, e\rangle : d \text{ and } e \text{ share exactly one of their parents}\}$$

Determine whether $R$ is:

**(i)** reflexive on the set of human beings
**(ii)** symmetric on the set of human beings
**(iii)** transitive on the set of human beings

Explain your answers.

# A straightforward and elementary example

(a) What is a binary relation?

A binary relation is a set of zero or more ordered pairs.

# A straightforward and elementary example

(b) $R = \{\langle d, e\rangle : d \text{ and } e \text{ share exactly one of their parents}\}$

**(i)** Is $R$ reflexive on the set of human beings? No.
I share two parents with myself, not one.

**(ii)** Is $R$ symmetric on the set of human beings? Yes.
If human beings $d$ and $e$ share exactly one parent, clearly $e$ and $d$—the very same people—share exactly one parent too.

**(iii)** Is $R$ transitive on the set of human beings? No.
For example, my maternal half-sister Rachel and I share exactly one parent, and me and my paternal half-sister Debby share exactly one parent, but Rachel and Debby share no parents.

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